Optimal quota for sector-specific immigration

by

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Abstract

Sectoral labor supply shortage is a cause of concern in many OECD countries and has raised support for immigration as a potential remedy. In this paper, we derive a general equilibrium model with overlapping generations, where natives require a compensating wage differential for working in one sector rather than in another. We identify price and wage effects of immigration on three different groups of natives: the young working in one of two sectors and the old. We determine the outcome of a majority vote on immigration into a given sector as well as the social optimum. The main findings are that i) the old determine the majority voting outcome of positive immigration into both sectors, if natives are not mobile across sectors, ii) the young determine the majority voting outcome of zero immigration into both sectors, if natives are mobile across sectors, iii) the social optimum is smaller than or equal to the majority voting outcome, and iv) sector-specific immigration is not always a substitute for native mobility across sectors.

Key words: immigration, political economy, welfare, sectoral mobility.

JEL codes: F22, J31, J61.

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1 Introduction

Immigration policy is one of the most pressing issues in developed countries that face large and growing numbers of migrants, and it is just as controversial. While immigration is opposed because of expected negative wage effects on native workers, it is also often promoted as a way to alleviate labor shortages in specific labor market segments, skill- or occupation-wise. In a situation of excess labor demand, an increase of labor supply via immigration is an alternative to an increase in prices to achieve labor market equilibrium. The existing literature on the labor market effects of immigration typically studies wage effects in a one-sector economy, but does not consider price effects of immigration (Card 2001, Borjas 2003, Ottaviano and Peri 2005). Recent exceptions are Cortes (2006), who estimates price effects of low-skilled immigration in the U.S. and Felbermayr and Kohler (2007), who derive wage and price effects of immigrants with heterogeneous skills, taking skill-specific quotas as given.

Immigration policies that aim to increase native welfare have to take into account both wage and price effects, as it is real wages, not nominal ones, which determine the overall welfare impact of immigration. Existing immigration policies reflect these concerns. A number of countries identify occupational shortages regularly and use these as criteria to favor or facilitate immigration.1 In Europe, a standard procedure is to subject potential immigrants to an employment test, where an employer needs to declare his need of the immigrant for a job that cannot be filled by any resident qualified candidate.2 In the United Kingdom, a sectors-based migrant worker scheme was introduced in 2002 for low-skill jobs in the sectors of hotels and catering, and food manufacturing.3 In Australia, potential immigrants in required occupations receive extra points in the immigrant selection process.4

In this paper, we derive optimal policies to select immigration by sector, taking into account both wage and price effects of immigration. We analyze a labor market that is segmented into two sectors. Inter-sectoral mobility is restricted because natives exhibit sector-specific work preferences: for given job characteristics, they require a compensating wage differential for working in one sector rather than in the other.5 6 In reality, job preferences are only one reason for wage differentials existing in the absence of productivity differences. Our model applies equally to restricted mobility due to geographic moving costs between sectors, costs associated with the loss of sector-specific human capital or the necessary acquisition of new human capital.7 Natives are heterogeneous in terms of the amount of their required

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1See OECD (2006).
2In nursing occupations, in particular, labor market shortages are already present in many countries and are likely to expand with the ageing of native populations. This is highlighted by heated political debates on the legal employment of immigrants in nursing occupations in Austria recently.
4Miller (1999).
5Klaver and Visser (1999) find for different sectors in the Dutch economy that their image is not good enough, at the going wage rate, to attract a sufficient number of workers, even if supply is abundant. Compare OECD (2003, p. 104).
6Borjas (2007) mentions in his blog the health sector in the UK as an example of a ‘low-wage ghetto’, which ‘drives natives into alternative, better-paid options and fulfills the prophecy that there are some jobs that natives just won’t do.’
7According to Zimmermann et al. (2007), there is evidence for regional and sectoral wage differentials within occupations, which suggests that mobility is insufficient even within a relatively homogeneous labor market (see DeNew and Schmidt 1994, Möller and Bellmann (1996) and Haisken, DeNew and Schmidt (1999) for Germany).
wage differential or moving cost. This is the main difference in our model between natives and migrants. Since we are interested in selective immigration policies targeted at specific sectors or occupations, we assume that natives and migrants are homogeneous within sectors.

The second key feature of our model is an overlapping generations structure. We derive optimal sector-specific immigration for three different groups of natives, the young working in one of two sectors, and the old. Intuitively, one would expect that old (retired) generations have a stronger preference for migration than young generations, since they do not experience potentially negative wage effects, but only positive price effects. Further, one would expect that old generations do not have a strong preference for migration to be specific to one or the other sector of the economy, while young generations should have such preferences, since their wages are directly affected by the sector-specific structure of the migration flow. We show that the old support immigration into both sectors, while the young oppose immigration into 'their' sector. However, the attitude of the young will be different, if they can move from one sector to the other as a response to a change in wage differentials due to immigration, given their sector-specific work preference. We derive the optimal amount of sector-specific immigration under two regimes, one where the sector choice of workers is fixed, and one where it is endogenous to migration.

Given that different groups are affected differently, what is the likely outcome on sector-specific immigration policies? We employ majority voting as a simple means of aggregating individual preferences and find that, in the case of fixed sector choice, immigration quota are strictly positive in both sectors. In the case of endogenous sector choice, immigration quota are likely to be smaller in both sectors, because while workers still oppose immigration into 'their' sector, they now experience a negative wage effect also from immigration into the other sector. Numerical simulations show that immigration quota are zero for a range of existing parameter value estimates. We then compare these politically determined quota to the quota that maximize social welfare. Socially optimal immigration quota are likely to be smaller in the case of fixed sector choice, because the majority voting outcome is determined by the preferences of the old, who do not take the negative effect of immigration on wages into account. In numerical simulations, we find that socially optimal immigration is zero under both fixed and endogenous sector choice for chosen parameter values.

Finally, we compare the welfare effects of immigration in the two regimes of fixed and endogenous sector choice of natives. In particular, one might expect that sector-specific immigration is a substitute, in terms of social welfare, for native mobility across sectors. This expectation is inherent in a number of recent policy proposals that suggest enhancing the mobility of native workers as an alternative to immigration when dealing with labor shortages. However, depending on relative labor supply in the two sectors, it is possible that the welfare effect of immigration is higher with than without native mobility. We identify conditions under which the substitutability result does, and does not, hold.

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8We could also think of the two sectors as a low- and a high-skill sector, with natives being heterogeneous with regard to their cost of investing in human capital.

9Compare Zimmermann et al. (2007), p.73. They suggest financial incentives such as tax exemptions for relocation or commuting costs as a way to increase inter-sectoral native mobility. The same idea was pronounced by the Austrian minister for economics and labor in 2007.
2 The model

Consider a two-country, two-period overlapping-generations economy. Every agent is born with one unit of inelastically supplied labor. Labor is perfectly substitutable within sectors. The labor endowments of countries I and II are $N > 0$ and $M > 0$, respectively, and constant over time.

2.1 Production

In country I, there are two production sectors. The output of sector $A (X_A)$ is non-tradable and hence can be consumed only in country I. The output of sector $B (X_B)$ is tradable and hence can be consumed in both countries. In country II, there is only one sector whose output ($X_C$) is tradable. The production in each sector is subject to constant-returns-to-scale technology:

$$X_i = L_i^\gamma_i K_i^{1-\gamma_i}, \quad \gamma_i \in (0, 1), \quad i \in \{A, B, C\} \tag{1}$$

where $L_i$ and $K_i$ denote labor and capital input used by sector $i$, respectively. Each sector is under perfect competition: the unit price of a production factor is equal to the value of its marginal product. Therefore, the sectoral wages are

$$w_i \equiv p_i \frac{\partial X_i}{\partial L_i} = p_i \gamma_i \left(\frac{K_i}{L_i}\right)^{1-\gamma_i} \tag{2}$$

where $p_i$ is the unit price of the output by sector $i$. We assume perfectly mobile capital such that there is only one interest rate that is equal to the value of the marginal product of capital in every sector, i.e., $\forall i$,

$$r \equiv p_i \frac{\partial X_i}{\partial K_i} = p_i (1 - \gamma_i) \left(\frac{L_i}{K_i}\right)^{\gamma_i} \tag{3}.$$

2.2 Consumption

Each agent in country I has the following inter-temporal utility function:

$$u_I(x) + \frac{1}{1+\delta} u_I(x) \tag{4}$$

where $\delta \geq 0$ is the common rate of discounting future consumption; $x (1) = (x_A (1), x_B (1), x_C (1))$ is the first-period consumption bundle; and $x (2) = (x_A (2), x_B (2), x_C (2))$ is the second-period consumption bundle. It implies that utility is invariant and additive over time. We assume $\partial u_I / \partial x_i > 0$ and $\partial^2 u_I / \partial^2 x_i < 0$. More specifically,

$$u_I(x) \equiv \alpha \ln x_A + \beta \ln x_B + (1 - \alpha - \beta) \ln x_C, \quad \alpha, \beta, \alpha + \beta \in (0, 1) \tag{5}.$$
The objective of the agent is to maximize the utility function subject to \( p(1) \cdot x(1) \leq w - s \) and \( p(2) \cdot x(2) \leq (1 + r)s \) where \( s \) is savings. Due to non-satiating utility and no bequest, the combined budget constraint is \( p(1) \cdot x(1) + p(2) \cdot x(2) / (1 + r) = w \).

We assume that the interest rate to the first-period savings is paid in the very beginning of the second period. The first-period savings are used as capital input in the first period, and the interest rate is determined via equation (3) in the same period.

For each agent in country II, we replace \( u_i \) in (4) by

\[
w_i(x) \equiv \theta \ln x_B + (1 - \theta) \ln x_C, \quad \theta \in (0, 1)
\]

because the output of sector \( A \) in country I is not tradable.

Accordingly, we obtain the following demand functions:

\[
x_A^1(1) = e \theta \frac{w_A(1)}{p_A(1)}, \quad x_A^2(1) = (1 - e) \alpha \frac{w_A(1)}{p_A(2)} (1 + r(1)) ,
\]

\[
x_B^1(1) = e \theta \frac{w_B(1)}{p_B(1)}, \quad x_B^2(1) = (1 - e) \beta \frac{w_B(1)}{p_B(2)} (1 + r(1)) ,
\]

\[
x_C^1(1) = e (1 - \alpha - \beta) \frac{w_A(1)}{p_C(1)}, \quad x_C^2(1) = (1 - e) (1 - \alpha - \beta) \frac{w_A(1)}{p_C(2)} (1 + r(1)) ,
\]

\[
x_A^1(2) = 0, \quad x_A^2(2) = 0,
\]

\[
x_B^1(2) = (1 - e) \theta \frac{w_B(1)}{p_B(2)} (1 + r(1)), \quad x_B^2(2) = (1 - e) \beta \frac{w_B(1)}{p_B(2)} (1 + r(1)) ,
\]

\[
x_C^1(2) = (1 - \theta) \frac{w_C(1)}{p_C(1)}, \quad x_C^2(2) = (1 - \theta) \frac{w_C(1)}{p_C(2)} (1 + r(1)) ,
\]

where \( e \equiv (1 + \delta)/(2 + \delta) \) and the superscript indicates the employment sector, e.g., \( x_A^2(1) \) is the first-period demand for the output of sector \( A \) by an agent who is employed in sector \( B \). In the second period, agents use their first-period savings plus interest for consumption.

### 2.3 Factor supply

The demand functions imply that individual savings \( s_i \) form a fixed fraction \( 1 - e \) of each agent’s labor income \( w_i \). Each agent inelastically supplies one unit of labor in the first period of life. Accordingly, in each period, total capital is equal to total savings given by

\[
\sum_i K_i = \sum_i s_i L_i = (1 - e) \sum_i w_i L_i .
\]

In country I, agents choose one of the two national sectors for work in the first life-period. Let \( \omega_h \in (-\infty, \infty) \) denote the wage differential between sectors \( A \) and \( B \) required by young agent \( h \) to work in
sector A. Young agent \( h \) chooses to work in sector A, if \( w_A - \omega_h > w_B \) and in sector B otherwise. We assume a continuous cumulative distribution function \( \Phi (\cdot) \) of young agents with respect to the required wage differential.\(^{11}\) Denote the wage differential by

\[
\omega \equiv w_A - w_B. \tag{8}
\]

Since young agent \( h \) with \( \omega_h < \omega \) chooses to work in sector A, \( \Phi (\omega) \) gives the fraction of country I’s young native population choosing to work in sector A.

We assume that \( M \) is very large compared to \( N \). Accordingly, the wage in country II is so low that country I can face a large number of country-II workers who want to migrate to either sector A or B. Country I can decide on the number of immigrant workers in sectors A and B. By doing so, the country decides on the number of workers in sector C residually, since we assume an inelastic supply of labor and full employment.

Accordingly, at given wage rates and a given number of migrants, labor supply in each sector is given by

\[
L_A = N \Phi(\omega) + M_A \geq 0 \tag{9}
\]

\[
L_B = N (1 - \Phi(\omega)) + M_B \geq 0 \tag{10}
\]

\[
L_C = M - M_A - M_B \geq 0 \tag{11}
\]

Let \( N \Phi(\omega) \equiv N_A \) and \( N (1 - \Phi(\omega)) \equiv N_B \) in the following.

### 2.4 Equilibrium

We set the supply of each sector’s output given by equation (1) equal to each sector’s demand expressed by the individual demand functions times the respective population sizes given by equations (9)-(11). In any given period \( t \), the following relationships hold:

\[
X_A(t) = e^\alpha \frac{w_A(t) L_A(t) + w_B(t) L_B(t)}{p_A(t)} + (1 - e^\alpha) \frac{w_A(t-1) L_A(t-1) + w_B(t-1) L_B(t-1)}{p_A(t)} (1 + r(t-1)) \tag{12}
\]

\[
X_B(t) = e^\beta \frac{w_A(t) L_A(t) + w_B(t) L_B(t)}{p_B(t)} + e^\gamma \frac{w_C(t) L_C(t)}{p_B(t)} + (1 - e^\beta) \frac{w_A(t-1) L_A(t-1) + w_B(t-1) L_B(t-1)}{p_B(t)} (1 + r(t-1))
+ (1 - e^\gamma) \frac{w_C(t-1) L_C(t-1)}{p_B(t)} (1 + r(t-1)) \tag{13}
\]

\(^{11}\)Let us assume the corresponding density function is non-degenerate, i.e. \( \phi(\cdot) > 0 \omega_h \).
\[ X_C(t) = e(1-\alpha-\beta) \frac{w_A(t) L_A(t) + w_B(t) L_B(t)}{p_C(t)} + e(1-\theta) \frac{w_C(t) L_C(t)}{p_C(t)} \]
\[ + (1-e)(1-\alpha-\beta) \frac{w_A(t-1) L_A(t-1) + w_B(t-1) L_B(t-1)}{p_C(t)} (1+r(t-1)) \]
\[ + (1-e)(1-\theta) \frac{w_C(t-1) L_C(t-1)}{p_C(t)} (1+r(t-1)) \]  

(14)

By substituting wages given by (2) into these and solving the system, we obtain the equilibrium prices:

\[ p_i(t) = \psi_i(t) \frac{1}{X_i(t)} \]  

(15)

where \( \psi_i(t) \) depends only on \( r \) and \( \psi_i \) in the previous period \( t-1 \). This in turn implies that the value of output in each sector \( p_i(t) X_i(t) \) is predetermined in any period: a change in labor supply in a given sector due to immigration changes the output and the price in that sector such that the value of output remains constant \( \psi_i(t) \).

**Lemma 1** Immigration affects neither the total amount of capital in the world, nor its distribution across sectors, nor the interest rate.

**Proof.** By substituting (15) into (3), we get

\[ \frac{K_i}{K_j} = \frac{\psi_i 1-\gamma_i}{\psi_j 1-\gamma_j}, \quad i, j \in \{A, B, C\}, \quad i \neq j \]  

(16)

for any given period. The capital ratio between sectors is thus predetermined. The resource constraint (7) implies, by using (2) and (15),

\[ \sum_i K_i = (1-e) \sum_i \gamma_i \psi_i \]  

(17)

for any given period, which shows that total capital in each period is a constant share of the sectoral output values \( \psi_i \) in that period, which, in turn, depend on the values of output in the previous period. This implies that total capital and labor supply in the initial period completely determine total capital in subsequent periods. ■

Since total capital only depends on the value of sectoral output, which is predetermined, the interest rate is fixed. An increase in \( L_A \) due to an increase in immigration would be completely offset by a decrease in \( p_A \) and, accordingly, in wages and savings. Immigration does not change total capital and, therefore, the interest rate, because while it increases the number of savers in country I, savings per capita fall due to the effect of a price decrease on the wage. \(^{13}\) Immigration increases the physical marginal product of capital, but not its value.

\[^{12}\text{The expression for } \psi_i(t) \text{ is too long for inclusion in the main text and is left to the appendix.}\]

\[^{13}\text{Note that this is true under our assumption that capital is made up of savings in the same period.}\]
2.5 Fixed sector choice

We now examine the outcome of a referendum on immigration into sectors A and B in country I. Let us assume for the moment that the sectoral choice of young natives is given and that none of the three groups of voters has an absolute majority.

**Proposition 1** Suppose the sectoral choice of young natives is fixed. In the absence of an absolute majority of any one group, majority voting on a pair of immigration quotas into the two sectors results in the amount of immigration that is preferred by the old,

\[
M_A = \frac{\alpha \gamma_A (M + N_B) - N_A [\beta \gamma_B + (1 - \alpha - \beta) \gamma_C]}{\alpha \gamma_A + \beta \gamma_B + (1 - \alpha - \beta) \gamma_C},
\]

\[
M_B = \frac{\beta \gamma_B (M + N_A) - N_B [\alpha \gamma_A + (1 - \alpha - \beta) \gamma_C]}{\alpha \gamma_A + \beta \gamma_B + (1 - \alpha - \beta) \gamma_C},
\]

given \(M_A > 0\) and \(M_B > 0\). \(M_A = 0\) and \(M_B = 0\) otherwise.

**Proof.** We derive individual indirect utility of young natives in country I by substituting the demand functions, wages (2) and prices (15) into the utility function (4). We have, for \(i \neq C\) and \(t = 1\),

\[
v_y(1) \equiv \alpha \ln \left( e^{\alpha w_i (1)} \right) + \beta \ln \left( e^{\beta w_i (1)} \right) + (1 - \alpha - \beta) \ln \left( e^{(1 - \alpha - \beta) \frac{w_i (1)}{p_C (1)}} \right)
\]

\[
+ \frac{1}{1 + \delta} \left[ \alpha \ln \left( (1 - e^{\alpha w_i (1)}) (1 + r (1)) \right) + \beta \ln \left( (1 - e^{\beta w_i (1)}) (1 + r (1)) \right) + (1 - \alpha - \beta) \ln \left( (1 - e^{(1 - \alpha - \beta) w_i (1)}) (1 + r (1)) \right) \right],
\]

where superscript \(y\) denotes young.

The indirect utility of a retired agent is simply the one-period lag of the fourth term in the above expression without the discounting:

\[
v_o(1) \equiv \alpha \ln \left( (1 - e^{\alpha w (0)}) (1 + r (0)) \right) + \beta \ln \left( (1 - e^{\beta w (0)}) (1 + r (0)) \right)
\]

\[
+ (1 - \alpha - \beta) \ln \left( (1 - e^{(1 - \alpha - \beta) w (0)}) (1 + r (0)) \right),
\]

where superscript \(o\) denotes old.\(^{14}\)

\(^{14}\)We omit the sector subscript from \(v\) and \(w\) in (19) because only the three output prices are affected by immigration in this expression, which enter utility in the same way regardless of whether an agent worked in sector A or B in the previous period.
For the first derivative of indirect utility of a young native in sector $A$ with respect to immigration into the sector, we get
\[ \frac{\partial v^y}{\partial M_A} (1) = \frac{1}{w_A (1)} \frac{dw_A (1)}{dM_A (1)} - \frac{\alpha}{p_A (1)} \frac{dp_A (1)}{dM_A (1)} - \frac{1 - \alpha - \beta}{p_C (1)} \frac{dp_C (1)}{dM_A (1)} + \frac{1}{(1 + \delta)w_A (1)} \frac{dw_A (1)}{dM_A (1)} \]
where the first and fourth terms are the wage effects. Note that the wage effect in the fourth term is felt in the second period of lifetime. The second term is the sector-$A$ price effect, while the third term is the sector-$C$ price effect. We can drop time indices and reduce the expression to
\[ \frac{\partial v^y}{\partial M_A} = -\frac{1}{e L_A} + \frac{\alpha \gamma_A}{L_A} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}. \tag{20} \]
The first derivatives of the indirect utility of young natives in sector $B$ and the old with respect to $M_A (1)$ are the same:
\[ \frac{\partial v^y}{\partial M_A} = \frac{\partial v^o}{\partial M_A} = \frac{\alpha \gamma_A}{L_A} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}. \tag{21} \]
That is, immigration into sector $A$ affects workers in sector $B$ and the old only via the price effects: a decrease in the sector-$A$ price and an increase in the sector-$C$ price. Therefore, the wage effect represented by the first term of (20) is missing in (21).

Analogously, regarding immigration into sector $B$, we have
\[ \frac{\partial v^y}{\partial M_B} = -\frac{1}{e L_B} + \frac{\beta \gamma_B}{L_B} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}, \tag{22} \]
and
\[ \frac{\partial v^o}{\partial M_B} = \frac{\partial v^o}{\partial M_B} = \frac{\beta \gamma_B}{L_B} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}. \tag{23} \]
Immigration into sector $A$ ($B$) affects utility of workers already in the sector in three ways, namely via (i) a decrease in the wage (first term on the right-hand side of (20) and (22)), (ii) a decrease in the sector-$A$ ($B$) price (second term) and (iii) an increase in the sector-$C$ price (third term). The total effect is negative because the negative wage effect dominates the positive sector-$A$ ($B$) price effect, i.e., $1/e > \alpha \gamma_A$ and $1/e > \beta \gamma_B$. Young natives in sector $A$ ($B$) therefore oppose immigration into their own sector. Their optimal amount of $M_A$ ($M_B$) is zero.

Immigration into sector $A$ ($B$) affects utility of young natives in the other sector and the old only via the price effects. (21) and (23) indicate a decrease in the sector-$A$ ($B$) price and an increase in the sector-$C$ price. The relative size of the price effects depends on the relative size of the native population in sector $A$ ($B$) and the foreign population $M$.\footnote{Assume that the population in country II is very large relative to the population in country I. Then, the positive price effect initially dominates the negative price effect. Young natives support immigration into the sector that is not their own, and the old support immigration into both sectors. With an increase in $M_A$ or $M_B$, this positive net effect of immigration decreases and eventually becomes negative.}
Young natives in sector $A$ choose $M_B$ by setting $M_A = 0$ and solving

$$\frac{\beta \gamma_B}{L_B} = \frac{(1 - \alpha - \beta) \gamma_C}{L_C}. \quad (24)$$

Young natives in sector $B$ choose $M_A$ by setting $M_B = 0$ and solving

$$\frac{\alpha \gamma_A}{L_A} = \frac{(1 - \alpha - \beta) \gamma_C}{L_C}. \quad (25)$$

Old natives choose $M_A$ and $M_B$ such that

$$\frac{\alpha \gamma_A}{L_A} = \frac{\beta \gamma_B}{L_B} = \frac{(1 - \alpha - \beta) \gamma_C}{L_C}. \quad (26)$$

By substituting (9)-(11) into these, we find the choice by young natives in sector $A$ is

$$\left( M_A^A, M_B^A \right) = \left( 0, \frac{\beta \gamma_B M - (1 - \alpha - \beta) \gamma_C N_B}{\beta \gamma_B + (1 - \alpha - \beta) \gamma_C} \right) \quad (27)$$

The choice by young natives in sector $B$ is

$$\left( M_A^B, M_B^B \right) = \left( \frac{\alpha \gamma_A M - (1 - \alpha - \beta) \gamma_C N_A}{\alpha \gamma_A + (1 - \alpha - \beta) \gamma_C}, 0 \right) \quad (28)$$

The choice by old natives is a pair of

$$M_A^o = \frac{\alpha \gamma_A [M + N_B] - [\beta \gamma_B + (1 - \alpha - \beta) \gamma_C] N_A}{\alpha \gamma_A + \beta \gamma_B + (1 - \alpha - \beta) \gamma_C} \quad (29)$$

and

$$M_B^o = \frac{\beta \gamma_B [M + N_A] - [\alpha \gamma_A + (1 - \alpha - \beta) \gamma_C] N_B}{\alpha \gamma_A + \beta \gamma_B + (1 - \alpha - \beta) \gamma_C} \quad (30)$$

Given $M_A^o > 0$ and $M_B^o > 0$, we have $M_B^o > M_A^o > M_A^A$ and $M_A^B > M_B^o > M_B^B$. Hence the median voter is elderly. ■

In our model, the young in both sectors oppose immigration into their respective sectors because such immigration depresses their wages: these negative effects dominate any positive price effect. However, they desire immigration into the other sector up to the point where the marginal utility from the decrease in that sector’s output price is equal to the marginal utility from the increase in the price of output produced in country II.

The old desire immigration into both sectors such that the marginal utility from the decrease in the price of both sectors’ output is equal to the marginal increase in the price of output produced in country II. They do not care about the wage effect of immigration. The pair of quotas chosen by the old represents
the median position because the young each form a majority with the old on immigration into the sector other than their own. The majority voting outcomes on immigration into sectors A and B is equal to $M_A^o$ and $M_B^o$ for $M_A^o > 0$ and $M_B^o > 0$.

### 2.6 Endogenous sector choice

In the following, we allow for the sector choice of young natives to respond to immigration. Immigration changes the wage differential between sectors A and B and, therefore, the distribution of natives across these sectors.

The equilibrium distribution of migrant and native workers across sectors A and B is determined by labor demand (2) and labor supply (10)-(11). Labor supply in sectors A and B is defined implicitly by the following two equations:

\[
F_A \equiv N \Phi \left( \frac{\psi_A \gamma_A}{L_A} - \frac{\psi_B \gamma_B}{L_B} \right) + M_A - L_A = 0 \tag{31}
\]

\[
F_B \equiv N \left( 1 - \Phi \left( \frac{\psi_A \gamma_A}{L_A} - \frac{\psi_B \gamma_B}{L_B} \right) \right) + M_B - L_B = 0 \tag{32}
\]

where we substituted for $\omega$ by using (8), (2) and (15).

**Lemma 2** Immigration into a given sector increases labor supply in both sectors A and B. But the increase in that given sector is larger when immigration enters into that sector than when it enters into the other sector.

**Proof.** Since

\[
\det \left( \begin{array}{cc} \frac{\partial F_A}{\partial L_A} & \frac{\partial F_A}{\partial L_B} \\ \frac{\partial F_B}{\partial L_A} & \frac{\partial F_B}{\partial L_B} \end{array} \right) = 1 + N \Phi(\omega) \left( \frac{\psi_A \gamma_A}{L_A^2} + \frac{\psi_B \gamma_B}{L_B^2} \right) \neq 0,
\]

11
we apply Cramer’s rule to the system (31)-(32) to get

\[
\frac{\partial L_A}{\partial M_A} = \frac{1 + N\Phi(\omega)\frac{\psi_A^2}{L_A} + \psi_B^2}{1 + N\Phi(\omega)\left(\frac{\psi_A^2}{L_A} + \frac{\psi_B^2}{L_B}\right)} \in (0,1)
\]

(33)

\[
\frac{\partial L_B}{\partial M_A} = \frac{N\Phi(\omega)\frac{\psi_A^2}{L_A}}{1 + N\Phi(\omega)\left(\frac{\psi_A^2}{L_A} + \frac{\psi_B^2}{L_B}\right)} \in (0,1)
\]

(34)

\[
\frac{\partial L_A}{\partial M_B} = \frac{N\Phi(\omega)\frac{\psi_A^2}{L_B}}{1 + N\Phi(\omega)\left(\frac{\psi_A^2}{L_A} + \frac{\psi_B^2}{L_B}\right)} \in (0,1)
\]

(35)

\[
\frac{\partial L_B}{\partial M_B} = \frac{1 + N\Phi(\omega)\frac{\psi_A^2}{L_B}}{1 + N\Phi(\omega)\left(\frac{\psi_A^2}{L_A} + \frac{\psi_B^2}{L_B}\right)} \in (0,1)
\]

(36)

These expressions imply that \(\frac{\partial L_A}{\partial M_A} > \frac{\partial L_A}{\partial M_B}\) and \(\frac{\partial L_B}{\partial M_A} > \frac{\partial L_B}{\partial M_B}\).

As a response to immigration into a given sector, some natives choose to switch. Immigration into sector A, for example, decreases the wage in that sector such that the wage differential becomes too small for some to compensate them for working in sector A. They choose sector B rather than sector A for employment. Immigration into sector B, on the other hand, increases the wage differential, such that more natives are now willing to work in sector A than before. Immigration into a given sector thus causes a movement of natives away from that sector, partially offsetting the initial wage decrease. Immigration is not totally offset by the sector change of natives. This is because, while immigration into a given sector decreases the wage in that sector, the ensuing sector switch by natives results in a wage decrease also in the other sector. As a result, labor supply increases in both sectors, and wages and prices decrease in both sectors.\(^{16}\)

We now derive the majority voting outcome for the case of endogenous sector choice.

**Proposition 2.** Assume natives can switch sector in response to immigration. Then, in the absence of an absolute majority of any one group, majority voting results in \(M^B_A\) and \(M^A_B\), where \(M^B_A\) is the amount of immigration into sector A that is preferred by workers in sector B and \(M^A_B\) is the amount of immigration into sector B that is preferred by workers in sector A.

**Proof.** Expressions (18) and (19) imply the following derivatives with respect to immigration into sector A:

\[
\frac{\partial v^B_A}{\partial M_A} = \left(\alpha\gamma_A - \frac{1}{e}\right)\frac{1}{L_A} \frac{\partial L_A}{\partial M_A} + \beta\gamma_B \frac{\partial L_B}{\partial M_A} - \left(1 - \alpha - \beta\right)\gamma_C \frac{1}{L_C}
\]

(37)

\(^{16}\)Note that the distribution of natives across sectors resulting from immigration into sector A is different from the one resulting from immigration into sector B. This is because the change in the wage differential between sectors A and B is different (it decreases with \(M_A\) and increases with \(M_B\)).
\[ \frac{\partial \psi^B}{\partial M_A} = \frac{\alpha A}{L_A} \frac{\partial L_A}{\partial M_A} \left( \beta B - \frac{1}{e} \right) \frac{1}{L_B} \frac{\partial L_B}{\partial M_A} - \frac{(1 - \alpha - \beta) \gamma C}{L_C} \]  

or, substituting using (33)-(36):

\[ \frac{\partial \psi^B}{\partial M_A} = \frac{\alpha A}{L_A} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} \left( \beta B - \frac{1}{e} \right) \frac{1}{L_B} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} - \frac{(1 - \alpha - \beta) \gamma C}{L_C} \]  

\[ \frac{\partial \psi^B}{\partial M_B} = \frac{\alpha A}{L_A} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} \left( \beta B - \frac{1}{e} \right) \frac{1}{L_B} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} - \frac{(1 - \alpha - \beta) \gamma C}{L_C} \]  

Analogously, with respect to immigration into sector B, we have

\[ \frac{\partial \psi^B}{\partial M_B} = \frac{\alpha A}{L_A} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} \left( \beta B - \frac{1}{e} \right) \frac{1}{L_B} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} - \frac{(1 - \alpha - \beta) \gamma C}{L_C} \]  

\[ \frac{\partial \psi^B}{\partial M_B} = \frac{\alpha A}{L_A} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} \left( \beta B - \frac{1}{e} \right) \frac{1}{L_B} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} - \frac{(1 - \alpha - \beta) \gamma C}{L_C} \]  

\[ \frac{\partial \psi^B}{\partial M_B} = \frac{\alpha A}{L_A} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} \left( \beta B - \frac{1}{e} \right) \frac{1}{L_B} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} - \frac{(1 - \alpha - \beta) \gamma C}{L_C} \]  

\[ \frac{\partial \psi^B}{\partial M_B} = \frac{\alpha A}{L_A} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} \left( \beta B - \frac{1}{e} \right) \frac{1}{L_B} \frac{1}{1 + N_A \frac{\psi A \gamma A}{L_A} + \frac{\psi B \gamma B}{L_B}} - \frac{(1 - \alpha - \beta) \gamma C}{L_C} \]  

We observe that, as a result of sectoral mobility, immigration into a given sector not only has a negative wage effect and positive price effect in that sector, but also a negative wage effect and a positive price effect in the other sector. Compared to the case of fixed sector choice, workers experience an additional positive price effect on goods produced in the other sector, represented by the second term in (40) and the first term in (44), by own-sector immigration. They also experience a negative wage effect and a
positive price effect in their own sector, represented by the second term in (41) and the first term in (43),
by immigration into the other sector. The old experience an additional positive price effect, represented
by the second term in (42) and the first term in (45).

The size of the price and wage effects in sectors A and B is smaller compared to the case of fixed sector
choice, because the change in sectoral labor supply due to immigration is smaller: with fixed sector
choice, the marginal immigrant increases sectoral labor supply by 1, \( \frac{\partial L_A}{\partial M_A} = 1 \) and \( \frac{\partial L_B}{\partial M_B} = 1 \), while
with endogenous sector choice, he increases sectoral labor supply by less than 1, \( 0 < \frac{\partial L_A}{\partial M_A} < 1 \) and
\( 0 < \frac{\partial L_B}{\partial M_B} < 1 \), because of the crowding-out effect on native sectoral labor supply. In other words, sectoral
mobility mitigates price and wage effects of immigration.

Optimal amounts of immigration into sectors A and B are the solutions to solving simultaneously for the
respective first-order conditions, i.e. for

---

young natives in sector A: \( (40) = 0 \) and \( (43) = 0 \)

young natives in sector B: \( (41) = 0 \) and \( (44) = 0 \)

old natives: \( (42) = 0 \) and \( (45) = 0 \),

---

subject to the lower and upper bound constraints \( M_A > 0 \), \( M_B > 0 \), \( M_A + M_B < M \).

We know that for the young, the lower constraints on immigration will always be binding, because for
any \( M_A \geq 0 \) and \( M_B \geq 0 \), marginal utility from immigration into workers' own sector is always smaller
than marginal utility from immigration into the other sector. Therefore, young natives in sector A choose
\( M_B \) by setting \( M_A = 0 \) and solving for \( (43)=0 \). Young natives in sector B choose \( M_A \) by setting \( M_B = 0 \)
and solving for \( (41)=0 \).

In sum, young natives in sector A choose \( (0, M_B^A) \), young natives in sector B choose \( (M_B^B, 0) \) and old
natives choose \( (M_A^o, M_B^o) \), where \( M_B^A, M_B^B, M_A^o \) and \( M_B^o \) are equal to the solutions to the first-order
conditions in (43), (41), (42) and (45), if positive and equal to zero, otherwise.\(^{17}\)

The outcome of a majority vote on immigration into a given sector is determined by the young. This is
because the old prefer a higher amount of immigration into each sector than both the young in sector
A or B, since their marginal utility is higher (compare (42) and (45) with (40), (41) and (43), (44),
respectively). The young always prefer zero immigration into their own sector, as argued above. It
follows that \( M_A^o \geq M_B^o \geq M_A^o \) and \( M_B^o \geq M_B^o \geq M_B^o \).\(^{18}\) That is, for \( M_B^B > 0 \), the young in sector B
determine immigration into sector A and for \( M_A^A > 0 \), the young in sector A determine immigration into
sector B. For \( M_A^o = 0 \) or \( M_B^o = 0 \), the young in both sectors form a majority on zero immigration in
sector A or B. Finally, for \( M_A^o = 0 \) or \( M_B^o = 0 \), every native votes for zero immigration into sector A or
B.

---

\(^{17}\)Note that the workers who switch sector as a response to immigration are indifferent as the marginal effect on their
utility is zero (compare section 3.2).

\(^{18}\)Weak inequality follows from the fact that immigration is subject to a lower limit of zero.
We report numerical solutions in Table 1 below to show the result of the majority voting outcome for reasonable parameter values. We find that, for the parameter values chosen, no positive solutions to $M^B_A$ or $M^A_B$ exist. In other words, there are no values $M_A > 0$ and $M_B > 0$ at which the positive price effects are dominating the negative effects on price C and workers’ wages. The young in both sectors will therefore vote against immigration not only into their own sector, but also into the other sector. The outcome of a majority vote is $M_A = 0$ and $M_B = 0$.

3 Social welfare analysis

In the previous section, we determined the outcome of a majority vote on the amount of sector-specific immigration. In the following, we determine the amount of immigration that is socially optimal, i.e. the amount chosen by a benevolent social planner who simultaneously determines immigration into sectors A and B.

In the following, we use as welfare criterion the sum of individual utilities in the standard utilitarian form:

$$W(M_i) = v^y_AN_A + v^y_BN_B + v^oN_o, \quad i=A,B.$$  \hspace{1cm} (46)

3.1 Fixed sector choice

Without sectoral mobility, the marginal social welfare effects of immigration into sectors A and B equal:

$$\frac{\partial W}{\partial M_A} = \frac{\partial v_A}{\partial M_A}N_A + \frac{\partial v_B}{\partial M_A}N_B + \frac{\partial v^o}{\partial M_A}N_o.$$  \hspace{1cm} (47)

and

$$\frac{\partial W}{\partial M_B} = \frac{\partial v_A}{\partial M_B}N_A + \frac{\partial v_B}{\partial M_B}N_B + \frac{\partial v^o}{\partial M_B}N_o.$$  \hspace{1cm} (48)

Using (20)-(23) to substitute in (47) and (48) gives

$$\frac{\partial W}{\partial M_A} = \left[-\frac{1}{eL_A} + \frac{\alpha \gamma_A}{L_A} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}\right]N_A + \left[\frac{\alpha \gamma_A}{L_A} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}\right](N_B + N_o)$$  \hspace{1cm} (49)

and

$$\frac{\partial W}{\partial M_B} = \left[\beta \gamma_B \frac{1}{L_B} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}\right](N_A + N_o) + \left[-\frac{1}{eL_B} + \frac{\beta \gamma_B}{L_B} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}\right]N_B.$$  \hspace{1cm} (50)

Immigration into sector A(B) has a positive price effect on good A(B) and a negative price effect on good
C for all three groups of voters, and it has a negative wage effect for the group of natives working in sector A(B).

In optimum, the marginal welfare of immigration into sector A via its effect on the wage and price in sector A (if positive) equals its (negative) marginal welfare via the effect on the price in sector C. Similarly, a positive marginal welfare of optimal immigration into sector B via its effect on the wage and price in sector B equals its negative marginal welfare via the effect on the price in sector C.

Proposition 3. Without sectoral mobility, socially optimal immigration \( (M_A^*, M_B^*) \) equals

\[
\begin{align*}
&M \left[ -\frac{1}{2}N_A + \alpha \gamma_A(N_A + N_B + N_o) \right] + (N_A + N_B + N_o)\left[ \alpha \gamma_A N_A - N_A(\beta \gamma_B - (1 - \alpha - \beta)\gamma_C)\right] \\
&- \frac{1}{2}(N_A + N_B) + \left[ \alpha \gamma_A + \beta \gamma_B + (1 - \alpha - \beta)\gamma_C\right](N_A + N_B + N_o),
\end{align*}
\]

\[
M \left[ -\frac{1}{2}N_B + \beta \gamma_B(N_A + N_B + N_o) \right] + (N_A + N_B + N_o)\left[ \beta \gamma_B N_A - N_B(\alpha \gamma_A - (1 - \alpha - \beta)\gamma_C)\right] \\
- \frac{1}{2}(N_A + N_B) + \left[ \alpha \gamma_A + \beta \gamma_B + (1 - \alpha - \beta)\gamma_C\right](N_A + N_B + N_o)
\]

for \((\alpha \gamma_A - \frac{1}{2})N_A + \alpha \gamma_A(N_B + N_o) > 0\) and \((\beta \gamma_B - \frac{1}{2})N_B + \beta \gamma_B(N_A + N_o) > 0\). It equals

\[
\left(0, M \left[ \beta \gamma_B N_A + (-\frac{1}{2} + \beta \gamma_B N_B) - (1 - \alpha - \beta)\gamma_C N_B(N_A + N_B + N_o) \right] \right)
\]

for \((\alpha \gamma_A - \frac{1}{2})N_A + \alpha \gamma_A(N_B + N_o) < 0\) and \((\beta \gamma_B - \frac{1}{2})N_B + \beta \gamma_B(N_A + N_o) > 0\). And it equals

\[
\left( M \left[ \alpha \gamma_A N_B + (-\frac{1}{2} + \alpha \gamma_A N_A) - (1 - \alpha - \beta)\gamma_C N_A(N_A + N_B + N_o) \right] \right)
\]

for \((\alpha \gamma_A - \frac{1}{2})N_A + \alpha \gamma_A(N_B + N_o) > 0\) and \((\beta \gamma_B - \frac{1}{2})N_B + \beta \gamma_B(N_A + N_o) < 0\), given \(M_A^* \geq 0\) and \(M_B^* \geq 0\), respectively. Socially optimal immigration into both sectors is zero, otherwise.

Proof. We simultaneously solve for \((49)=0\) and \((50)=0\) with respect to \(M_A\) and \(M_B\) subject to their lower and upper bounds \(M_A \geq 0\), \(M_B \geq 0\) and \(M_A + M_B \leq M\). Given \(M_A \geq 0\), \(M_B \geq 0\) and \(M_A + M_B \leq M\), the marginal welfare effect of immigration into sector A is positive (and, therefore, \(M_A > 0\)) if and only if \((\alpha \gamma_A - \frac{1}{2})N_A + \alpha \gamma_A N_B > 0\). Analogously, the marginal welfare effect of immigration into sector B is positive (and, therefore, \(M_B > 0\)) if and only if \((\beta \gamma_B - \frac{1}{2})N_B + \beta \gamma_B N_A > 0\), as can be seen by transforming \((49)\) and \((50)\).

Socially optimal amounts of immigration \(M_A^*\) and \(M_B^*\) are likely to be smaller than the respective majority voting outcomes, which correspond to the preferences of the old \(M_A^0\) and \(M_B^0\), because the old do not take the negative effect of immigration on wages into account. Simulation results reported in Table 1 show that, for chosen parameter values, socially optimal immigration into both sectors A and B is zero in the case of fixed sector choice.
3.2 Endogenous sector choice

With sectoral mobility, the marginal social welfare effects of immigration into sectors A and B equal:

\[
\frac{\partial W}{\partial M_A} = \frac{\partial v_A}{\partial M_A} N \Phi(w_A - w_B) + v_A N \Phi \left( \frac{\partial w_A}{\partial M_A} - \frac{\partial w_B}{\partial M_A} \right) \\
+ \frac{\partial v_B}{\partial M_A} N [1 - \Phi(w_A - w_B)] - v_B N \Phi \left( \frac{\partial w_A}{\partial M_A} - \frac{\partial w_B}{\partial M_A} \right) \\
+ \frac{\partial v_o}{\partial M_A} N_o. \tag{51}
\]

and

\[
\frac{\partial W}{\partial M_B} = \frac{\partial v_A}{\partial M_B} N \Phi(w_A - w_B) + v_A N \Phi \left( \frac{\partial w_A}{\partial M_B} - \frac{\partial w_B}{\partial M_B} \right) \\
+ \frac{\partial v_B}{\partial M_B} N [1 - \Phi(w_A - w_B)] - v_B N \Phi \left( \frac{\partial w_A}{\partial M_B} - \frac{\partial w_B}{\partial M_B} \right) \\
+ \frac{\partial v_o}{\partial M_B} N_o. \tag{52}
\]

Using (40)-(45) to substitute in (51) and (52) gives

\[
\frac{\partial W}{\partial M_A} = \left[ \frac{\alpha \gamma_A}{L_A} \frac{\partial L_A}{\partial M_A} + \frac{\beta \gamma_B}{L_B} \frac{\partial L_B}{\partial M_A} - \left( 1 - \alpha - \beta \right) \frac{\gamma_C}{L_C} \right] (N_A + N_B + N_o) \\
- \frac{1}{e} \left[ \frac{N_A}{L_A} \frac{\partial L_A}{\partial M_A} + \frac{N_B}{L_B} \frac{\partial L_B}{\partial M_A} \right] \\
+ (v_A - v_B) N \Phi \left( -\psi \frac{\gamma_A}{L_A^2} \frac{\partial L_A}{\partial M_A} + \psi \frac{\gamma_B}{L_B^2} \frac{\partial L_B}{\partial M_A} \right) \tag{53}
\]

and

\[
\frac{\partial W}{\partial M_B} = \left[ \frac{\alpha \gamma_A}{L_A} \frac{\partial L_A}{\partial M_B} + \frac{\beta \gamma_B}{L_B} \frac{\partial L_B}{\partial M_B} - \left( 1 - \alpha - \beta \right) \frac{\gamma_C}{L_C} \right] (N_A + N_B + N_o) \\
- \frac{1}{e} \left[ \frac{N_A}{L_A} \frac{\partial L_A}{\partial M_B} + \frac{N_B}{L_B} \frac{\partial L_B}{\partial M_B} \right] \\
+ (v_A - v_B) N \Phi \left( -\psi \frac{\gamma_A}{L_A^2} \frac{\partial L_A}{\partial M_B} + \psi \frac{\gamma_B}{L_B^2} \frac{\partial L_B}{\partial M_B} \right), \tag{54}
\]

where we can further substitute for the change in total sectoral labor supply due to immigration using (33)-(36).

The first square bracket gives the price effects of immigration: positive effects on prices A and B and a negative effect on price C, affecting all three groups of voters: the young in sectors A and B as well as the old. The second term gives the negative wage effects on the young in sectors A and B. The third term gives the effect on utility of the young that switch sector due to immigration, for given wages.\footnote{The third term comes from the fact that we compute the derivative of the products of group-specific indirect utilities with group sizes, both of which depend on immigration. Applying the product rule, we get the derivative of indirect utilities times group size (terms 1 and 2) plus the derivatives of group sizes times indirect utilities (term 3).} We can
show that the third square bracket is negative - that is, immigration into sector A reduces wage A more strongly than wage B. As a consequence, natives switch from sector A into sector B. Now, given equal job conditions in both sectors, utility is higher in sector A than in sector B, since \( w_A > w_B \). However, we have assumed that job conditions in A are considered worse than in B, such that workers require a compensating wage differential for working in sector A. Therefore, at the margin, utility derived from wages adjusted for job conditions does not change for those who switch sector.\(^{20}\) The marginal effect of immigration for natives who switch sector is zero.

In order to solve for socially optimal immigration in the case of endogenous sector choice, we again simultaneously solve for (53)=0 and (54)=0 with respect to \( M_A \) and \( M_B \) subject to their lower bounds \( M_A \geq 0, M_B \geq 0 \) and upper bound \( M_A + M_B \leq M \). The solutions to \( M_A, M_B \) are quadratic due to the quadratic terms in (53) and (54). We report numerical solutions below. We find that, for reasonable parameter values, no positive solutions to \( M_A \) or \( M_B \) exist. In other words, there are no values \( M_A > 0 \) and \( M_B > 0 \) for which any positive net price effects of immigration are dominating the negative wage effects of immigration on society. Hence, for chosen parameter values, socially optimal immigration is \( M_A = 0 \) and \( M_B = 0 \) also in the case of endogenous sector choice.

3.3 Immigration and native mobility across sectors

Policymakers typically suggest two ways to solve the problem of excess labor demand: 1) the enhancement of sectoral mobility of natives\(^{21}\) or 2) the immigration of foreign workers. The two policies are often perceived to be mutually exclusive: undertaking one policy is supposed to make the other one redundant. This would imply that immigration is socially more desirable in an immobile than in a mobile society.

In the following, we show that whether this is true or not depends on the size of native and immigrant working populations.

**Proposition 4.** Immigration into sector A(B) is socially more desirable when natives are mobile between sectors than when they are not, if \( \frac{\alpha_{\gamma A}}{\beta_{\gamma B}} < \frac{L_A}{L_B} < \frac{N_A}{N_B} \) \( (\frac{\beta_{\gamma B}}{\alpha_{\gamma A}} < \frac{L_B}{L_A} < \frac{N_B}{N_A}) \).

**Proof.** In order to compare the welfare effect of immigration into sector A with and without sectoral mobility, we compare (49) and (53). Sufficient conditions for the welfare effect to be higher with mobility than without are that i) the positive price effects are larger with mobility than without and ii) the negative wage effects are smaller with mobility than without.

\(^{20}\) \( v_A(x^{A+z}) = v_B(x^B) \), where \( x \) is the consumption good bundle purchased with money and expressed in terms of sector-specific wages and the prices, and \( z \) is the compensating variation for working in sector A compared with sector B. Compare Rosen (1986).

\(^{21}\) If switching sector comes at a cost, a change in the sectoral wage differential will not prompt as many natives to switch, as it would otherwise do. A policy of enhancing native sectoral mobility could consist of reducing this cost. We compare the two extreme cases of no mobility versus mobility restricted only by heterogeneous job preferences for the purpose of illustration.
First, the positive price effects are larger with mobility than without, if

\[
\frac{\alpha \gamma_A}{L_A} \frac{\partial L_A}{\partial M_A} + \frac{\beta \gamma_B}{L_B} \frac{\partial L_B}{\partial M_A} > \frac{\alpha \gamma_A}{L_A}
\]  

(55)

or, transforming using (33) and (34), if

\[
\frac{\alpha \gamma_A}{\beta \gamma_B} < \frac{L_A}{L_B}.
\]

(56)

With sectoral mobility, immigration into sector A decreases prices not only in sector A but also in sector B. For the price effects to be larger with than without sectoral mobility, the ratio of labor supply in sector A relative to labor supply in sector B must be large enough.

Second, the negative wage effects are smaller with mobility than without, if

\[
\frac{1}{\epsilon} \left[ \frac{1}{L_A} \frac{\partial L_A}{\partial M_A} N_A + \frac{1}{L_B} \frac{\partial L_B}{\partial M_A} N_B \right] < \frac{1}{\epsilon} \frac{L_A}{L_B} N_A
\]

(57)

or, transforming using (33) and (34), if

\[
\frac{L_A}{L_B} < \frac{N_A}{N_B}.
\]

(58)

With sector mobility, immigration into sector A decreases wages not only in sector A but also in sector B. For the wage effects to be smaller with than without sectoral mobility, the ratio of labor supply in sector A relative to labor supply in sector B must be small enough.

In sum,

\[
\frac{\alpha \gamma_A}{\beta \gamma_B} < \frac{L_A}{L_B} < \frac{N_A}{N_B}
\]

(59)

is a sufficient condition for immigration into sector A to have a larger positive effect on social welfare under sectoral mobility than under no sectoral mobility.

Analogously for immigration into sector B.

4 Simulation of immigration quotas

Optimal immigration quota derived above depend on the parameter values of our stylized model. In the case of endogenous sector choice, the expressions for optimal immigration are complex enough to best be derived numerically. We therefore use existing parameter estimates to provide a sense of the magnitude of optimal immigration into sectors, both for the groups of young and old, as well as for society overall.
This way, we can also easily compare the outcome of a majority vote with the social optimum. Since our stylized model is one of two countries, one home and one foreign, we think of immigration quotas as quotas for one (major) foreign country.

### 4.1 Parameter choice

For the relative size of the native working population $N_A + N_B$ and the foreign working population $M$ that comprises potential immigrants, we choose a ratio of 1 to 2. This way, we take account of the fact that the population of potential migrants, even if only from one country, can potentially be quite large and significantly outnumber native workers. We assume that slightly more workers are in the tradables sector B than in the non-tradables sector A and use $N_A = 0.4$ and $N_B = 0.6$. The relative size of the old within the native population is set to $N_o = 0.9$ since, according to ILO (2007) statistics, the share of the retired is almost equal to that of the working population in a typical OECD country.

For spending shares in consumption, we use estimates of the shares of spending on tradables and non-tradables in a typical OECD country from the Penn World Table. For domestic demand, we use $\alpha = 0.5$ for the non-tradable good produced in sector A and $\beta = 0.25$ for the tradable good produced in sector B, which implies $1 - \alpha - \beta = 0.25$ for the imported good produced in sector C. For foreign demand, we use $\theta = 0.5$ for the good imported from sector B, which implies $1 - \theta = 0.5$ for the good produced in sector C. For wage shares in production, we use estimates for a typical OECD country from ILO (2000). We set $\gamma_A = 0.7$, $\gamma_B = 0.7$ and $\gamma_C = 0.7$ in the three sectors A, B, C.

We choose a discount rate of 0.5 that corresponds to a yearly discount rate of 0.02 for the length of 20 years and a consumption rate of 0.6. For the world interest rate $r$, we use 0.15, as Kohler and Felbermayr (2007). We calibrate $p_A$ and, as implied by relative prices, $p_B$ and $p_C$ as well as capital stocks $K_A$, $K_B$ and $K_C$ according to a previous-period interest rate $r_{t-1}$ of 0.15, equal to the current interest rate.

We also show simulations for different values of the size of the native labor force, the labor intensity in domestic production, spending shares in domestic consumption and the output value in the previous period, which determines the value of output in subsequent periods according to (15) together with (60)-(62).

Table 2 summarizes the baseline parameter values described above and reports corresponding equilibrium values of the model.

### 4.2 Simulation results

Table 1 reports simulation results on immigration quotas for no sectoral mobility (panel a) and sectoral mobility (panel b). For each case, we show group-specific quotas as well as the social optimum for immigration into both sectors A and B. For no sectoral mobility, we find that old natives occupy the median position, as stated in Proposition 1. They prefer more immigration into a given sector than young natives working in that sector, but less immigration than young natives working in the other sector. Their preferred amount of immigration is strictly positive, whereas socially optimal immigration is zero for all chosen parameter values. Majority voting therefore results in immigration that is too high from a social
point of view.

For sectoral mobility, we find that the old still vote for strictly positive immigration into both sectors. Their optimal amounts of immigration are even higher than in the case of no sectoral mobility, which is due to the fact that the sum of the two positive price effects for the old is larger in utility terms than the single positive price effect derived from immigration without sectoral mobility.22 The young in both sectors still vote against immigration into their own sector, because for given parameter values, the larger positive price effects from immigration are still not large enough to cover the negative wage effect. Besides, they now also vote against immigration into the other sector, because the negative effect on their wage due to sectoral crowding-out is not compensated by the increase in positive price effects. The majority voting outcome in the case of sectoral mobility therefore is zero and equal to the social optimum, which is also zero.

In columns 2-5 of the table, we show results for variations of a single parameter at a time, so that they can be compared with results for the baseline in column 1. In column 2, the native work force in sector B is smaller: \( N_B = 0.5 \). We can see that, as a result, old natives and young natives in sector A (in case of no sectoral mobility) prefer more immigration into sector B. The reason is that the marginal effect of immigration into sector B on the price in that sector is now larger. Since the old and the young in sector A choose immigration into sector B such that marginal price effects in all sectors are equal, optimal immigration into sector B is higher. Note also that optimal immigration into sector A for the old is smaller in the case with no sectoral mobility, because optimality requires that the marginal price effects in sector A and B are the same (both are larger). With sectoral mobility, optimal immigration into sector A for the old is slightly higher as well, because of the spill-over effect of immigration into a given sector on the price in the other sector caused by workers who switch.

In column 3, we choose a higher wage share in sector A, \( \gamma_A = 0.8 \). In this case, the effect of a price decrease in sector A is larger, and optimal immigration into sector A increases for both the old as well as the young in sector B. As a consequence, optimal immigration into sector B decreases for the old, who equate marginal price effects in A and B, for no sectoral mobility. For sectoral mobility, an increase in \( \gamma_A \) increases utility derived from immigration into both sectors, due to spill-over effects. Since the marginal effect from immigration into sector A is larger, however, optimal immigration into sector B decreases.

We also consider an increase in the home bias in consumption, in the form of a higher domestic spending share on sector A: \( \alpha = 0.6 \) in column 4. As a result, the marginal effect of a decrease in price A increases relative to the marginal effect of an increase in price C. Optimal sector-A immigration for the old and, in case of no sectoral mobility, the young in sector B increases and is even higher than in column 3. Due to the decrease in the marginal effect on price C, optimal immigration into sector B for the young in sector A now also increases in the case of no sectoral mobility. It remains constant for the old, who compensate fully by an increase in preferred immigration into sector A.

\[ \text{Note that the negative effect on the price of the imported good is the same.} \]
Finally, we show results for an increase in the value of previous-period output in sector B $\psi_{Bt-1} = 2.17$. We can see that results for no sectoral mobility remain the same, as marginal effects of current immigration are independent of previous-period output values. However, in the case of sectoral mobility, a change in $\psi_{Bt-1}$ changes the effect of immigration on total sectoral labor supply, because it changes $\psi_{Bt}$ and $\psi_{At}$.

For chosen parameter values, both $\psi_{Bt}$ and $\psi_{At}$ increase, and immigration into a given sector increases labor supply in that sector less strongly than before. In other words, sectoral crowding-out increases, which in turn increases the positive price effects and, therefore, optimal immigration for the old increases.

5 Conclusion

We determine the outcome of a majority vote on immigration into sectors as well as the socially optimal amount, assuming that natives require a compensating wage differential for working in one sector rather than in another. We focus on immigrants who are selected to serve as substitutes for natives in a given sector, mapping situations of sectoral labor supply shortages that are present in many OECD countries. We identify both the wage and the price effects of such immigration in an overlapping generations model with three different groups of natives: young natives working in sector A, young natives working in sector B and the old. We find that the young are against immigration into their own sector, because the negative wage effect dominates the positive price effect. The old and the young in the other sector, who are affected by immigration only via price effects, but not via wage effects, support immigration and determine the outcome of the majority vote. As a result, a majority supports a strictly positive amount of immigration into both sectors.

We then allow for the sector choice of natives to change endogenously with immigration. In this case, optimal amounts of immigration depend on parameter values. We calibrate model parameters according to existing data and estimates and run a number of simulations to determine immigration quota outcomes. We find that, while young natives still prefer zero immigration into their own sector, they now also prefer zero immigration into the other sector. This is because immigration into a given sector now has a negative wage effect in both sectors, due to some natives switching sector. This wage effect via crowding-out, though smaller, still dominates positive price effects for plausible parameter values. The old now prefer a larger amount of immigration into both sectors, because of an increase in the positive price effects. The young in both sectors form a majority and vote for zero immigration into both sectors.

Further, we find that, for the range of parameter values used, socially optimal immigration into sectors A and B is zero both without and with sectoral mobility. Therefore, the majority voting outcome is too high in the case of no sectoral mobility, but optimal in the case of sectoral mobility from a social point of view.

\footnote{Note that this could be due to a change in initial capital or labor supply, according to (15) together with (60)-(62).}
\footnote{See (33)-(36) together with (60)-(62) in the Appendix.}
Appendix: Sectoral output values

In Section 2, we derived sectoral equilibrium prices $p_{it}$ as functions of the constants $\psi_{it}$ and sector outputs $X_{it}$, $i=$A, B, C in period $t$:

$$p_{it} = \frac{\psi_{it}}{X_{it}}.$$  \hfill (15')

We can easily see that the values of sectoral outputs are equal to the constants, expressions of which are given by the following:

$$\psi_{At} = z \left[ (\gamma_A \psi_{At-1} + \gamma_B \psi_{Bt-1}) \alpha (1 - e^{\gamma_C (1 - \theta)}) + \gamma_C \psi_{Ct-1} \alpha e^{\gamma_B \theta} \right]$$ \hfill (60)

$$\psi_{Bt} = z \left[ (\gamma_A \psi_{At-1} + \gamma_B \psi_{Bt-1}) (\beta (1 - e^{\gamma_C}) + e^{\gamma_C \theta (1 - \alpha)} + \gamma_C \psi_{Ct-1} (1 - e^{\gamma_A \alpha} \theta) \right]$$ \hfill (61)

$$\psi_{Ct} = z \left[ (\gamma_A \psi_{At-1} + \gamma_B \psi_{Bt-1}) (1 - \alpha - \beta) + \gamma_C \psi_{Ct-1} [(1 - \alpha e^{\gamma_A} - \beta e^{\gamma_B}) (1 - \theta) + e^{\gamma_B \theta} (1 - \alpha - \beta)] \right],$$ \hfill (62)

where

$$z = \frac{1 + r_{t-1}}{(2 + \delta) [(1 - e^{\alpha \gamma_A} - e^{\beta \gamma_B}) (1 - (1 - \theta) e^{\gamma_C}) - (1 - \alpha - \beta) e^{2 \gamma_B \gamma_C \theta}].}$$

As the constants and, therefore, sectoral output values, only depend on previous-period output values and exogenous parameters, they are exogenous in any given period and determined by initial sectoral output values $\psi_{A0}$, $\psi_{B0}$ and $\psi_{C0}$. 

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Table 1: Results on Immigration Quotas

<table>
<thead>
<tr>
<th>(1) Baseline</th>
<th>(2) Smaller native labor force</th>
<th>(3) Higher wage share in sector A</th>
<th>(4) Higher spending share on sector A</th>
<th>(5) Higher initial value of output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Immigration into sector A</td>
<td>Group-specific optimum</td>
<td>Workers in sector A</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Workers in sector B</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Old natives</td>
<td>1.1</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>Social optimum</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Immigration into sector B</td>
<td>Group-specific optimum</td>
<td>Workers in sector A</td>
<td>0.7</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Workers in sector B</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Old natives</td>
<td>0.15</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>Social optimum</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(a): No sectoral mobility

<table>
<thead>
<tr>
<th>(b): Sectoral mobility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Immigration into sector A</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Immigration into sector B</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Note: In column (2), the native labor force in B is $N_B = 0.5$. In column (3), the wage share in sector A is $\gamma_A = 0.8$. In column (4), the spending share of natives on goods produced in sector A is $\alpha = 0.6$. In column (5), the previous-period value of output in sector B is $\psi_{Bt-1} = 2.17$. 
Table 2: Benchmark - Calibration and Equilibrium

<table>
<thead>
<tr>
<th>Household utility per period: Cobb-Douglas</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameters</strong></td>
</tr>
<tr>
<td>$u_I = \alpha \ln x_A + \beta \ln x_B + (1 - \alpha - \beta) \ln x_C$</td>
</tr>
<tr>
<td>good produced in sector A</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
</tr>
<tr>
<td>$X_A^d = 0.5015$</td>
</tr>
<tr>
<td>$p_A = 0.8478$</td>
</tr>
<tr>
<td>good produced in sector B</td>
</tr>
<tr>
<td>$\beta = 0.25$</td>
</tr>
<tr>
<td>$X_B^d = 0.2650$</td>
</tr>
<tr>
<td>$p_B = 0.8024$</td>
</tr>
<tr>
<td>good produced in sector C</td>
</tr>
<tr>
<td>$1 - \alpha - \beta = 0.25$</td>
</tr>
<tr>
<td>$X_C^d = 0.6155$</td>
</tr>
<tr>
<td>$p_C = 0.3454$</td>
</tr>
<tr>
<td><strong>Foreign</strong></td>
</tr>
<tr>
<td>$u_{II} = \theta \ln x_B + (1 - \theta) \ln x_C$</td>
</tr>
<tr>
<td>good produced in sector B</td>
</tr>
<tr>
<td>$\theta = 0.5$</td>
</tr>
<tr>
<td>$X_B^d = 0.4698$</td>
</tr>
<tr>
<td>good produced in sector C</td>
</tr>
<tr>
<td>$1 - \theta = 0.5$</td>
</tr>
<tr>
<td>$X_C^d = 1.0913$</td>
</tr>
</tbody>
</table>

Technology: Cobb-Douglas

| Good produced in sector A                   |
| $X_A = L_A^\gamma_A K_A^{1-\gamma_A}$     |
| Wage share                                 |
| $\gamma_A = 0.7$                           |
| $X_A^s = 0.5015$                           |
| $p_A = 0.8478$                             |
| Good produced in sector B                   |
| $X_B = L_B^\gamma_B K_B^{1-\gamma_B}$     |
| Wage share                                 |
| $\gamma_B = 0.7$                           |
| $X_B^s = 0.7348$                           |
| $p_B = 0.8024$                             |
| Good produced in sector C                   |
| $X_C = L_C^\gamma_C K_C^{1-\gamma_C}$     |
| Wage share                                 |
| $\gamma_C = 0.7$                           |
| $X_C^s = 1.7068$                           |
| $p_C = 0.3454$                             |

Labor endowment and allocation

| Home | $N_A = 0.4$ | $w_A = 0.7442$ |
| Home | $N_B = 0.6$ | $w_B = 0.6879$ |
| Home | $N_o = 0.9$ |

Foreign

| Workers in sector C | $M = 2$ | $w_C = 0.2063$ |

Capital endowment and allocation

| Home | $\epsilon = 0.6$ (\(\delta = 0.5\)) |
| Home | $p_A X_A^\prime(K_A) = r$ |
| Home | $K_A = 0.8505$ |
| Home | $r = 0.15$ |
| Home | $p_B X_B^\prime(K_B) = r$ |
| Home | $K_B = 1.1792$ |
| Home | $r = 0.15$ |
| Home | $p_C X_C^\prime(K_C) = r$ |
| Home | $K_C = 1.1792$ |
| Home | $r = 0.15$ |

Note: We assume that the native sectoral population and the interest rate remain unchanged from the previous period, and that $p_{At-1}=1$, $p_{Bt-1}=1.5$, $p_{Ct-1}=0.8$. Then, $\psi_{At-1}=0.5383$, $\psi_{Bt-1}=1.4412$ and $\psi_{Ct-1}=1.9570$. Implied values of sectoral output in period $t$ are $\psi_{At}=0.4252$, $\psi_{Bt}=0.5896$, $\psi_{Ct}=0.5896$. 

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