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The bequest tax as long-term care insurance

by

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Abstract

I study a model of a representative individual who has a motive for leaving bequests and is at risk of needing long-term care in old age. I assume - as is typical for OECD countries - that the individual is not fully insured against this risk. Moreover, at realization the individual is unable to adapt labor supply or consumption; then expenditures for long-term care result in a one-to-one reduction of the estate. In this situation a tax on bequests provides insurance and its introduction causes a smaller deadweight loss than an income or consumption tax. I also characterize the optimal tax and transfer system in this model.

Keywords: Estate tax; long-term care insurance

JEL classification: H21, H24, I13

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1 Introduction

The tax on bequests or inheritances is a much contested issue in tax policy. It exists in a number of OECD countries such as the US, France and Germany, but was repealed in others such as Canada, Sweden and Austria. Advocates of this tax typically refer to its redistributive role (increasing "equality of opportunity"),[1] whereas opponents stress its distorting effect on savings and capital formation.[2]

In the standard optimal-taxation model, which allows an assessment of the welfare consequences of taxes, the result by Atkinson and Stiglitz (1976) tells us that in addition to an optimally chosen tax on labor income there is no role for indirect taxation - thus also not for a bequest tax; not even for redistributive reasons if individuals differ in their ability to earn income only.[3] If the fact that leaving bequests creates a positive external effect (for the donee, in addition to the utility it creates for the donor) is taken into account a subsidy for bequests turns out to be optimal (Blumkin and Sadka 2003, Farhi and Werning 2010). As shown in Brunner and Pech (2012a, 2012b), the idea that a tax on bequests should be imposed for redistributive reasons can be studied in an extended model, which accounts for the fact that due to wealth transfers over generations a second distinguishing characteristic (in addition to earning ability) of individuals arises, namely differences in received inheritances. If these and earning abilities are positively correlated, a bequest tax indeed increases the scope for redistribution.[4]

The results mentioned so far are derived under the assumption that individuals leave bequests deliberately, being motivated by joy-of-giving or by pure altruism (dynastic preferences). As is well-known, a further category of bequests are unintended bequests, which occur if for some reason individuals did not fully annuitize their wealth and die before it

[1] Gale and Slemrod 2001 show that in the US the estate tax is indeed a much more progressive tax than the income tax.

[2] Other arguments against the bequest tax emphasize its potential to impair the continuation of small businesses (see, however, Grossmann and Strulik 2010) or the inappropriatenes of using the moment of death as a cause for collecting a tax.

[3] This is literally true if preferences are weakly separable between consumption and labor (see also Kaplow 2001). Otherwise arguments following Corlett-Hague (1953) as well as distributive aspects may apply, which are, however, difficult to fix empirically (Deaton 1981).

[4] Cremer et al. (2001, 2003) and Boadway et al. (2000) show that if bequests are considered unobservable, taxes on capital income or on commodities can be justified by this argument.

is used up.[5] In this case, the bequest tax does not distort their behavior and is, thus, a first-best tax instrument. Moreover, it can also be viewed as providing some insurance against leaving unintended bequests (given insufficient annuitization, see Blumkin and Sadka 2003).

In the present paper I consider intended bequests, and I argue that a tax on these also makes sense as an insurance instrument. I formulate a model where a representative individual lives for two periods, works in the first and consumes in both. Moreover, she has a joy-of-giving motive for leaving bequests to her descendants, and there is a risk that she will be in need for long-term care (LTC) in the second period. As was mentioned before (Pauly 1990, see also Meier 1998), the amount an individual leaves as bequests is strongly connected with that risk: typically, at the time when the need for LTC becomes manifest - in old age - the individual is unable to adapt her behavior with respect to labor supply and consumption. This justifies the assumption employed in the model that expenses for LTC result in a one-to-one reduction of the estate left to her descendants.

Obviously, a first-best solution for the individual would be to have complete insurance. However, in accordance with reality I assume that the individual does not buy full insurance against LTC expenses and has to bear (part of) them. I show that in this situation a proportional bequest tax effectively provides partial insurance against the reduction of the estate by LTC expenses and is, thus, a preferable instrument compared to the income or consumption tax. If the tax rate on bequests is differentiated with respect to the severity of LTC need, then this result applies for the tax on bequests in the case of lowest LTC expenditures (largest bequests), while the opposite holds for the tax on bequests in the case of highest LTC expenditures (lowest bequests).

I also characterize the optimal tax and transfer system in this model, when tax revenues are used for the financing of LTC subsidies and a publicly provided good. It turns out that in the optimum the former do not cover the expenses completely, because of the excess burden of tax financing.

In a further section I extend the model and discuss the possibility that not only bequests

[5] For a typology of bequest motives see Cremer and Pestieau (2006) or Kopczuk (2010).

but also second-period consumption can be adapted after the realization of the LTC need. Then the consumption tax raises a different amount depending on the extent of LTC need and provides, thus, some insurance. But its beneficial effect is likely to be smaller than that of the bequest tax, because the tax base of the latter varies more with the state of LTC need.

In another extension I show that the advantage of the bequest tax over other taxes remains valid in case of dynastic (or altruistic) preferences. The latter mean that descendant utility is explicitly included in the parents' preferences, which provides a motive to leaving bequests. It turns out that the marginal deadweight loss of the respective taxes and the advantage of the bequest tax can be described in the just the same way as with a joy-of giving-motive. However, if the government's objective includes descendant utility separately, in addition to parent utility (what is sometimes called double counting), then from am intertemporal social-welfare perspective the positive effect of a bequest tax is lower compared to what the consideration of parent welfare alone suggests.

The assumption of no private insurance, wich is essential for the results, is justified by the observation that indeed in industrialized countries the market for private LTC insurance is very small and people buy far less insurance contracts than one might suspect.[6] Still, the reason for this fact is not finally established. Brown and Finkelstein (2007) find substantial load factors (between 18 and 51 cents per dollar) that make LTC insurance less attractive, but they also provide evidence that these supply factors cannot fully explain the small size of the market. On the demand side, one may argue that individuals rely – at least to some extent – on care provided by family members, which helps to avoid (too large) monetary expenditures so that the estate is not (drastically) reduced. An important further rationale for not purchasing (complete) insurance probably is the existence of publicly (e.g., by Medicaid in the US) financed LTC itself.

In the present paper I do not concentrate on the question of why the private insurance market is so underdeveloped. I just take the fact as given that most individuals do not buy

[6] On average, private LTC insurance covers less than two percent of total LTC expenditures in OECD countries; its share is highest in the US and Japan with 7% and 5%, respectively (OECD 2011, ch. 8). See also Brown and Finkelstein 2007, Pauly 1990, among others.

LTC insurance, and even if they do, the contract does not provide full insurance against the expenditure risk, but only rather limited coverage.[7] I show that in this situation imposing a bequest tax has a first-order advantage compared to an income or a consumption tax, irrespective of the use of the tax revenues. It should be emphasized that the argument indeed rests on the non-existence of *private* full insurance. Any kind of *public* insurance is financed by distorting taxes and for these the result stating the superiority of bequest-tax financing applies.[8]

In Section 2 the basic model is presented. In Section 3 the marginal deadweight loss of a proportional bequest tax is shown to be lower than that of an income or consumption tax. In Section 4 this kind of comparison is performed for a bequest tax with rates contingent on the realization of LTC expenditure. Section 5 provides a characterization of the optimal tax and transfer system. Section 6 deals with two extensions, in the first the case that consumption can be adapted is discussed, while in the second the joy-of-giving motive is related to the altruistic motive, where parents have dynastic preferences. Section 7 concludes.

2 The Model

I consider a representative individual who lives for two periods. Her consumption in the periods 1 and 2 is denoted by c and d, respectively, and she works l units of time in period 1. Moreover, at the end of period 2 she may leave bequests b, motivated by joy of giving. Her preferences can be described by the utility function $u(c, d, l) + v(b)$, with strictly concave u (increasing in c and d, decreasing in l) and v (increasing), where I assume, for simplicity, that bequests enter additively. Let s denote saving in period 1, w the wage rate, t the tax rate on labor income and τ the tax rate on consumption, then the budget constraint for period 1 reads as

$$c(1+\tau) + s \leq wl(1-t). \tag{1}$$

[7] According to Brown and Finkelstein (2007), typically purchased insurance policies "tend to cover one-third or less of the long-term care expenditure risk".

[8] For instance, in Germany there exists an explicit social LTC insurance with contributions proportional to income. Hence the finding of this paper justifies the imposition of a bequest tax to raise public revenues.

In period 2, the individual may end up in need for long-term care. The associated expenses x_i can assume three possible values, where $x_0 \equiv 0$ describes the case of no LTC need, while x_1 and x_2 with $0 < x_1 < x_2$ refer to states of increasing severity of disability. Let $\pi_i > 0, i = 0, 1, 2$, with $\pi_0 + \pi_1 + \pi_2 = 1$ denote the probabilities of these outcomes.

As mentioned in the Introduction, I assume - in accordance with reality - that the need for LTC typically arises at a moment in time when the person is unable to adapt her consumption decision nor her working decision. Hence LTC expenses reduce the amount of bequests she leaves to her descendants. On the other hand, the individual may receive social assistance $a_i \geq 0, i = 1, 2$, depending on the extent of need for care.[9] Define, in addition, $a_0 \equiv 0$ and let $b_i, i = 0, 1, 2$ denote bequests in the three possible situations. Moreover, bequests may be subject to a proportional tax σ, then for each realization of x_i the corresponding budget constraint of the individual in period 2 is (with r denoting the rate of interest)

$$d(1+\tau) + x_i - a_i + b_i(1+\sigma) \leq s(1+r), \qquad i = 0, 1, 2. \tag{2}$$

By using (1) and (2) in equality form one can eliminate s and obtains

$$c(1+\tau) + \frac{d(1+\tau)}{1+r} + \frac{b_0(1+\sigma)}{1+r} \leq wl(1-t), \tag{3}$$

$$b_i = b_0 - \frac{x_i - a_i}{1+\sigma}, i = 1, 2. \tag{4}$$

Condition (4) states that, as mentioned above, LTC expenses minus social assistance reduce the amount left to the descendants; the reason is that the individual cannot react by adapting other variables any more. I assume in the following that $0 < x_1 - a_1 \leq x_2 - a_2$ (that is, social assistance does not overcompensate higher LTC expenses) and, as a consequence, $b_0 > b_1 \geq b_2$. (Should the latter inequality not hold, one could simply reindex the cases 1 and 2.) The decision of the individual results from maximization of expected utility

$$u(c, d, l) + \sum_{i=0}^{2} \pi_i v(b_i), \tag{5}$$

[9] Note that the realization x_i is assumed to be observable, which allows a_i being contingent on x_i.

subject to (3) and (4) and to the nonnegativity conditions $c, d, b_2 \geq 0$. I assume an interior solution (that is, $c, d, b_2 > 0$), substitute for b_1 and b_2 using (4) in the objective function and derive the first-order conditions with respect to b_0 (subindices denote partial derivatives, λ is the Langrangean multiplier associated with (3)):

$$u_c - \lambda(1+\tau) = 0, \tag{6}$$

$$u_d - \lambda\frac{1+\tau}{1+r} = 0, \tag{7}$$

$$u_l + \lambda w(1-t) = 0, \tag{8}$$

$$\sum_{i=0}^{2} \pi_i v'(b_i) - \lambda\frac{1+\sigma}{1+r} = 0. \tag{9}$$

By eliminating λ from (6) - (8) the familiar marginal-rate conditions for consumption and labor supply are derived: $-u_l/u_c = w(1-t)/(1+\tau)$, $u_c/u_d = 1+r$, while from (9) and (6) one obtains the condition relating the expected marginal utility of bequests to the marginal utility of consumption: $\sum_{i=0}^{2} \pi_i v'(b_i)/u_c = (1+\sigma)/((1+\tau)(1+r))$.

3 Comparing the marginal deadweight loss

Let $U(t, \tau, \sigma, a_1, a_2)$ be the indirect utility function, that is, the optimal value of (5), subject to the constraints (3) and (4), for given tax rates t, τ and σ and for given values of the social assistance a_1 and a_2. I assume a benevolent government that wants to impose taxes in such a way that indirect utility is as large as possible. The question is which taxes it should use to attain this goal. In a first step, the answer is given by comparing the welfare effect of a marginal increase of either tax, ignoring the revenue requirement for the moment. More precisely, I compute the marginal deadweight loss of each of these taxes as the ratio of the marginal utility loss to the marginal additional revenue, created by a marginal tax increase.

Applying the Envelope Theorem, the respective marginal utility losses caused by t, τ and σ are given by:

$$\frac{\partial U}{\partial t} = -\lambda w l, \tag{10}$$

$$\frac{\partial U}{\partial \tau} = -\lambda(c + \frac{d}{1+r}), \tag{11}$$

$$\frac{\partial U}{\partial \sigma} = \sum_{i=0}^{2} \pi_i v'(b_i) \frac{x_i - a_i}{(1+\sigma)^2} - \lambda \frac{b_0}{1+r}, \tag{12}$$

while the marginal revenue effect follows from the formula for public revenues (discounted to period 1) $R = twl + \tau\tilde{c} + \sigma\bar{b}/(1+r)$, (where $\tilde{c} \equiv c + d/(1+r)$ and $\bar{b}$ denotes expected bequests $\sum_{i=0}^{2} \pi_i b_i$) as

$$\frac{\partial R}{\partial t} = w(l + t\frac{\partial l}{\partial t}) + \tau\frac{\partial \tilde{c}}{\partial t} + \sigma\frac{\partial \bar{b}}{\partial t}\frac{1}{1+r}, \tag{13}$$

$$\frac{\partial R}{\partial \tau} = \tilde{c} + \tau\frac{\partial \tilde{c}}{\partial \tau} + tw\frac{\partial l}{\partial \tau} + \sigma\frac{\partial \bar{b}}{\partial \tau}\frac{1}{1+r}, \tag{14}$$

$$\frac{\partial R}{\partial \sigma} = (\bar{b} + \sigma\frac{\partial \bar{b}}{\partial \sigma})\frac{1}{1+r} + tw\frac{\partial l}{\partial \sigma} + \tau\frac{\partial \tilde{c}}{\partial \sigma}. \tag{15}$$

Using (10) - (12) together with (9), the absolute value of the respective marginal deadweight loss, defined as $m_j \equiv -(\partial U/\partial j)/(\partial R/\partial j), j = t, \tau, \sigma$ of the three taxes, is found as

$$m_t = \sum_{i=0}^{2} \pi_i v'(b_i) \frac{1+r}{1+\sigma} \frac{wl}{\partial R/\partial t}, \tag{16}$$

$$m_\tau = \sum_{i=0}^{2} \pi_i v'(b_i) \frac{1+r}{1+\sigma} \frac{\tilde{c}}{\partial R/\partial \tau}, \tag{17}$$

$$m_\sigma = \sum_{i=0}^{2} \pi_i b_i v'(b_i) \frac{1}{1+\sigma} \frac{1}{\partial R/\partial \sigma}, \tag{18}$$

where I have used (4) to eliminate $x_i - a_i$ in (12) to get

$$\frac{\partial U}{\partial \sigma} = -\sum_{i=0}^{2} \pi_i b_i v'(b_i)/(1+\sigma). \tag{19}$$

In view of (13) - (18), in order to prove $m_\sigma < m_t, m_\tau$ at $\sigma = t = \tau = 0$ one needs to show that

$$\sum_{i=0}^{2} \frac{\pi_i b_i}{\bar{b}} v'(b_i) < \sum_{i=0}^{2} \pi_i v'(b_i). \tag{20}$$

Next observe that the sum $\sum_{i=0}^{2} \pi_i b_i/\bar{b} = 1$, thus the $\pi_i b_i/\bar{b}, i = 0, 1, 2$ are probability weights like the $\pi_i, i = 0, 1, 2$. Moreover, the ratio $b_i/\bar{b}$ is increasing with b_i, thus it is large for small values of $v'(b_i)$. As a consequence, the weighted sum on the LHS of (20) is indeed smaller than the weighted sum on the RHS, which implies our first result:[10]

Assume that the individual is not fully insured against the long-term care risk. Then the marginal deadweight loss of the introduction of a tax σ on bequests is smaller than the marginal deadweight loss of the introduction of a tax t on labor income or of a tax τ on consumption.

This result justifies a positive tax rate on bequests for efficiency reasons. It confirms the intuitive idea that a bequest tax allows a differentiated treatment of the individual according to the severity of her need for LTC care. By imposing a higher tax payment in case of less LTC need, that is, of higher bequests left to the descendants, it provides some insurance against the LTC risk, which neither the tax on labor income nor the tax on consumption does. Therefore, a first-order difference in the associated marginal deadweight loss occurs.

One may ask whether this conclusion can be extended to the situation with positive tax rates. Indeed, in view of the partial-market result that the deadweight loss increases quadratically with the tax rate, one expects that also in case of $t, \tau > 0$ the introduction of a bequest tax creates a lower marginal deadweight loss than an increase of any of the other taxes. Considering the formulas (16) - (18) as well as (20), one observes that in the present model the answer depends on how the expressions $wl/(\partial R/\partial t)$, $\tilde{c}/(\partial R/\partial \tau)$ and $1/(\partial R/\partial \sigma)$ develop with increasing t and τ, where still $\sigma = 0$. The relation $m_\sigma < m_t, m_\tau$ continues to hold if the first two expressions are larger than 1 for positive t and τ, but the last is not. One can see from (13) - (15) that the first two involve own and cross price effects of t and τ, while the third (for $\sigma = 0$) only involves cross-price effects of σ. This gives us a clear result for a specific type of preferences:

Assume that preferences are such that the cross price effects of σ are zero, while the own and cross price effects of t, τ on consumption and labor supply are non-positive. Then the above result that the marginal deadweight loss of the bequest tax is lower than that

[10] To see this more formally, note that the transition from the weights π_i to the weights $\pi_i b_i/\bar{b}$ can be decomposed into two steps. In the first step, π_2 is reduced to $\widehat{\pi}_2 \equiv \pi_2 b_2/\bar{b}$ (note $b_2 < \bar{b}$) and π_1 is increased to $\widehat{\pi}_1 \equiv \pi_1 + (\pi_2 - \widehat{\pi}_2)$, π_0 remains unaffected, $\widehat{\pi}_0 \equiv \pi_0$. As $v'(b_1) \leq v'(b_2)$, applying the weights $\widehat{\pi}_i$ instead of π_i certainly does not increase the weighted sum of the $v'(b_i), i = 0, 1, 2$. Next, increasing π_0 to $\pi_0 b_0/\bar{b}$ (note $b_0 > \bar{b}$) and decreasing $\widehat{\pi}_1$ accordingly (leaving $\widehat{\pi}_2 = \pi_2 b_2/\bar{b}$ unchanged), decreases the weighted sum because $v'(b_0) < v'(b_1)$.

of any other tax also holds in case of $t, \tau > 0$, $\sigma = 0$.

It is shown in Appendix A that quasilinear preferences fulfill the above assumption. Though it is not guaranteed in general that $m_\sigma < m_t, m_\tau$, this inequality clearly continues to hold as long as own-price reactions on tax revenues dominate cross-price reactions.

4 Differentiated tax rates on bequests

I generally assume that the extent of LTC need (the realization of x_i) is observable, therefore the government can design the tax on bequests in such a way that the rate depends on the realization. I briefly analyze this possibility, which can be described by introducing different rates σ_i imposed in the three outcomes $i = 0, 1, 2$. This means that in the model of Section 2 the σ_i replace σ in the constraints (2), $i = 0, 1, 2$; σ_0 occurs instead of σ in (3), while (4) becomes

$$b_i = b_0 \frac{1+\sigma_0}{1+\sigma_i} - \frac{x_i - a_i}{1+\sigma_i}, i = 1, 2. \tag{21}$$

As a consequence, the first-order condition (9) now reads as

$$\sum_{i=0}^{2} \pi_i v'(b_i) \frac{1+\sigma_0}{1+\sigma_i} - \lambda \frac{1+\sigma_0}{1+r} = 0. \tag{22}$$

The marginal utility loss caused by σ_i is

$$\frac{\partial U}{\partial \sigma_0} = \sum_{k=1}^{2} \pi_k v'(b_k) \frac{b_0}{1+\sigma_k} - \lambda \frac{b_0}{1+r}, \tag{23}$$

$$\frac{\partial U}{\partial \sigma_i} = \pi_i v'(b_i) \frac{-b_0(1+\sigma_0) + x_i - a_i}{(1+\sigma_i)^2}, i = 1, 2. \tag{24}$$

Using (21) and (22), (23) and (24) can be simplified to

$$\frac{\partial U}{\partial \sigma_i} = -\pi_i v'(b_i) \frac{b_i}{1+\sigma_i}, i = 0, 1, 2. \tag{25}$$

The marginal revenue of σ_i is $\partial R/\partial\sigma_i = (\pi_i b_i + \sum_{k=0}^{2} \sigma_k \pi_k \partial b_k/\partial\sigma_i)/(1+r) + tw\partial l/\partial\sigma_i + \tau\partial\tilde{c}/\partial\sigma_i$. Thus, at $\sigma_k = 0, k = 0,1,2$ and $t = \tau = 0$, the comparison $m_{\sigma_i} \lesseqgtr m_t, m_\tau$ results in[11]

$$v'(b_i) \lesseqgtr \sum_{k=0}^{2} \pi_k v'(b_k), i = 0,1,2. \tag{26}$$

For $i = 0$ (the case of largest bequests) the LHS of (26) is certainly lower than the RHS, while the opposite holds for $i = 2$ (lowest bequests). For $i = 1$ the inequality is undetermined in general. Clearly, differentiated tax rates allow a still better insurance against LTC need than a single tax rate. With respect to their welfare effects we find:

The introduction of a tax which affects bequests in case of no (of largest) private LTC expenditures causes a lower (larger) marginal deadweight loss than the introduction of any other tax.

5 The optimal tax and transfer system

In this section I provide a more comprehensive analysis of how a welfare-maximizing government should set the taxes and transfers. The revenue side consists, as in the previous section, of taxes on income, on consumption and on bequests, and I return to the case of a single rate σ on the latter. The expenditure side comprises the social assistance in cases of LTC needs, and for the sake of completeness I also introduce a publicly provided good, denoted by g, which is consumed by the individual in the first period. I assume that g enters additively via some strictly concave and increasing function $h(g)$, hence the utility function now reads as $u(c,d,l) + v(b) + h(g)$.

A first result can be derived from the individual budget constraints (3), (4):

(i) Either the income tax or the consumption tax is redundant, given that the social assistance can be adapted appropriately.

To see this, divide (3) by $1+\tau$, change t to t' such that $1-t' = (1-t)/(1+\tau)$ and σ to σ' such that $1+\sigma' = (1+\sigma)/(1+\tau)$ and, finally, let a_i' be such that $(x_i - a_i')/(1+\sigma') =$

[11] Note that in (16) and (17) σ_i occurs instead of σ, and in (13) and (14) the terms $\sigma(\partial\bar{b}/\partial t)$ and $\sigma(\partial\bar{b}/\partial\tau)$, respectively, have to be replaced by analogous sum expressions involving σ_i. They can be neglected at $\sigma_i = 0$.

$(x_i - a_i)/(1+\sigma), i = 1, 2$. Then one arrives at a tax system t', σ' (without a consumption tax) and transfers a_i', which is equivalent to the original system consisting of $t, \tau, \sigma, a_1, a_2$. The procedure to find an equivalent tax system without an income tax is similar.

In the following I consider a tax system without a consumption tax. Let $U(t, \sigma, a_1, a_2, g)$ be the indirect utility function depending on the instruments of the government. Its problem is to

$$\text{maximize} \qquad U(t, \sigma, a_1, a_2, g), \tag{27}$$

$$\text{s.t.} \qquad twl + \sigma \frac{\bar{b}}{1+r} \geq \pi_1 \frac{a_1}{1+r} + \pi_2 \frac{a_2}{1+r} + g. \tag{28}$$

With γ denoting the Langrangean variable associated with (28), the first-order conditions read as

$$\frac{\partial U}{\partial t} + \gamma(wl + tw\frac{\partial l}{\partial t} + \sigma \frac{\partial \bar{b}}{\partial t} \frac{1}{1+r}) = 0, \tag{29}$$

$$\frac{\partial U}{\partial \sigma} + \gamma(tw\frac{\partial l}{\partial \sigma} + \frac{\bar{b}}{1+r} + \sigma \frac{\partial \bar{b}}{\partial \sigma} \frac{1}{1+r}) = 0, \tag{30}$$

$$\frac{\partial U}{\partial a_1} - \gamma \frac{\pi_1}{1+r} = 0, \tag{31}$$

$$\frac{\partial U}{\partial a_2} - \gamma \frac{\pi_2}{1+r} = 0, \tag{32}$$

$$\frac{\partial U}{\partial g} - \gamma = 0. \tag{33}$$

From (29) - (33) the following properties of an optimal tax and transfer system can be derived:

(ii) The transfers a_1 and a_2 should be determined such that bequests are identical in both cases where private LTC expenditures arise, that is, $x_1 - a_1 = x_2 - a_2$, and thus $b_1 = b_2$.

This is a consequence of (31) and (32) and the fact that application of the Envelope Theorem to the individual maximization problem gives $\partial U/\partial a_i = \pi_i v'(b_i)/(1+\sigma)$, therefore $v'(b_1) = v'(b_2)$ follows.

(iii) The amount of g should be chosen such that the marginal rate of substitution between bequests in the states of positive private LTC expenditures and the publicly

provided good, $v'(b_i)/h'(g)$, is equal (in absolute value) to the discounted price of bequests, $(1+\sigma)/(1+r)$; note that the price of g is one.

This result is implied by $\partial U/\partial g = h'$ together with (33) and $\gamma = v'(b_i)/(1+\sigma)$ (from (31) and (32)).

(iv) In general, the social assistance should not provide full insurance, that is $a_i < x_i, i = 1, 2$.

Assume on the contrary that $a_1 = x_1, a_2 = x_2$, thus $b_1 = b_2 = b_0 = \bar{b}$. From (12) and (30) one obtains $b_0(-\lambda + \gamma)/(1+r) + \gamma(tw\partial l/\partial\sigma + \sigma\partial b_0/\partial\sigma) = 0$. Moreover, (9) implies $\lambda = v'(b_0)(1+r)/(1+\sigma) = \gamma$, where the latter equality follows from (31) and $\partial U/\partial a_i = v'(b_0)/(1+\sigma)$. Thus, one arrives at a contradiction, given that the expression $tw\partial l/\partial\sigma + \sigma\partial b_0/\partial\sigma$ is unequal to zero; the latter is negative in case of a negatively sloped demand curve for bequests and a non-positive reaction of labor supply l on the tax rate σ.

(v) Assume that preferences are quasilinear, then in an optimal tax and transfer system the tax rate on bequests should be positive.

This final result confirms the finding of the previous section. To prove it assume $\sigma = 0$ and use (19) and $\gamma = v'(b_1)(1+r)/(1+\sigma)$ (from (31)) to see that the LHS of (30) has the same sign as $\sum_{i=0}^{2} \pi_i b_i v'(b_i)/\bar{b} - v'(b_1)$. (20) and $\sum_{i=0}^{2} \pi_i v'(b_i) < v'(b_1)$ (note that there is no full insurance) imply that the LHS of (30) is positive, not zero. This contradicts $\sigma = 0$.

6 Extensions

6.1 Adjustable consumption

The results so far were derived from the assumption that the individual is unable to adapt any economic variable other than bequests. Though this seems indeed to be the realistic approach (Pauly 1990), I now briefly discuss the case that in the model of Sections 2 and 3 the individual can react with old-age consumption as well, when the need for LTC expenses is realized. To account for this, let by $d_i, i = 0, 1, 2$ denote old-age consumption depending on the realization of LTC need. The objective function of the individual now reads as

$$\sum_{i=0}^{2} \pi_i[u(c, d_i, l) + v(b_i)], \tag{34}$$

and in the second-period budget constraint (2) d is replaced by d_i:

$$d_i(1 + \tau) + x_i - a_i + b_i(1 + \sigma) \leq s(1 + r), i = 0, 1, 2. \tag{35}$$

The first-order conditions for the maximization of (34) subject to (1) and (35) with respect to c, l, s, d_i, b_i are written as ($\nu_i, i = 0, 1, 2$ are the Lagrangean variables to (35), λ refers to (1), as before)

$$\sum_{i=0}^{2} \pi_i u_c(c, d_i, l) - \lambda(1 + \tau) = 0, \tag{36}$$

$$\sum_{i=0}^{2} \pi_i u_l(c, d_i, l) + \lambda w(1 - t) = 0 \tag{37}$$

$$-\lambda + \sum_{i=0}^{2} \nu_i(1 + r) = 0, \tag{38}$$

$$\pi_i u_{d_i}(c, d_i, l) - \nu_i(1 + \tau) = 0, \qquad i = 0, 1, 2, \tag{39}$$

$$\pi_i v'(b_i) - \nu_i(1 + \sigma) = 0, \qquad i = 0, 1, 2. \tag{40}$$

The marginal utility loss is described by (10) and

$$\frac{\partial U}{\partial \tau} = -\lambda c - \sum_{i=0}^{2} \nu_i d_i, \tag{41}$$

$$\frac{\partial U}{\partial \sigma} = -\sum_{i=0}^{2} \nu_i b_i. \tag{42}$$

With revenues $R = twl + \tau c + \tau \bar{d}/(1 + r) + \sigma \bar{b}/(1 + r)$, where $\bar{d} \equiv \sum_{i=0}^{2} \pi_i d_i$ is expected old-age consumption, the comparison $m_\sigma \lesseqgtr m_\tau$, at $t = \tau = \sigma = 0$, can be written as (after substituting for λ and ν_i from the first-order conditions)

$$\frac{\sum_{i=0}^{2} \pi_i v'(b_i) b_i}{\bar{b}} \lesseqgtr \frac{\sum_{i=0}^{2} \pi_i v'(b_i)(c + \frac{d_i}{1+r})}{c + \frac{\bar{d}}{1+r}}. \tag{43}$$

Obviously, if old-age consumption reacts to the realization of LTC expenses, then the consumption tax also provides some insurance by raising a larger amount of tax in the good state than in the bad state. (Note that the first-order conditions imply $u_{d_i} = v'(b_i)$ at $\tau = \sigma = 0$.)

Whether the bequest tax or the consumption tax is preferable (in terms of a lower marginal deadweight loss) depends on the probability weigths $\pi_i b_i/\bar{b}$ and $\pi_i(c + d_i/(1 + r))/(c+\bar{d}/(1+r))$, respectively, which are applied to the marginal utilities $v'(b_i)$, increasing with i. Intuitively, the insurance effect is larger for that tax whose base shows a larger variation of realizations relative to the expected value. To see this, assume that for the intermediate ($i = 1$) realization the ratio to the mean is the same for both taxes. Then the LHS in (43) is lower than the RHS if $b_0/\bar{b} > (c + d_0/(1 + r))/(c + \bar{d}/(1 + r))$, because this implies $b_2/\bar{b} < (c+d_2/(1+r))/(c+\bar{d}/(1+r))$, and $v'(b_0) < v'(b_2)$ holds generally. Clearly, the precise comparison is quite complex and depends on the curvature of v, among others.

On the other hand, from considerations similar to those leading to (43) the definite result $m_\sigma < m_t$ follows; the reason being that - almost by definition - the income tax is imposed on an economic variable which cannot be adapted as soon as an individual needs LTC. To summarize:

If second-period consumption is adapted to the extent of LTC need, the introduction of a consumption tax causes a lower marginal deadweight loss than the introduction of an income tax. Whether it is lower compared to the introduction of a bequest tax depends on the variation of the respective tax base relative to its mean.

Certainly, the more plausible case is that the variation, relative to the mean, of bequests is larger than that of lifetime consumption; hence the bequest tax is the preferable instrument. Remember that the potential of the consumption tax to provide insurance comes from the fact that, after the removal of uncertainty regarding the LTC need, an individual chooses a higher (lower) consumption if the good (bad) state has occured. Then she experiences a differentiated tax burden for the rest of her lifetime, but only after the realization of LTC expenditures is known and consumption has been adapted. In reality, the point of time when the need for LTC becomes manifest is itself a random variable which is very likely to assume a value near the end of life.[12] Thus, only a small share of total lifetime consumption can be adapted. In other words, the model of Section 3, where the time of LTC realization is fixed and occurs at the very end of life, seems to be a better approximation to reality than the model of this section, where realization is fixed to occur at the beginning of the retirement period.

Finally, it should be mentioned that the positive effect of the consumption tax would be larger if old-age consumption could be taxed at a specific rate. It is straightforward to see that in this case the comparison with the effect of the bequest tax looks similar to (43), but without young-age consumption c appearing in the numerator and denominator on the RHS; the relative variation would be larger. However, in general it is infeasible to tax consumption at a different rate depending on the age of the consumer. A more convenient instrument could be to introduce a specific tax on those goods which are typically consumed by older individuals to a larger extent.

6.2 Dynastic preferences

Now I extend the model of Sections 2 and 3 and study the bequest tax as an instrument for LTC insurance, if its effect on the next generation is explicitly included. The intention is to relate the analysis of this paper to the standard discussion of the estate tax (Blumkin and Sadka 2003, among others). Moreover, I want to work out whether the results change if bequests are motivated by pure altruism instead of joy of giving.

A representative individual of the second generation also lives for two periods. Her consumption in both periods and labor supply in the first period are denoted by c^n, d^n and l^n, respectively. In her second period of life she is assumed to receive (stochastic) inheritances $b_i, i = 0, 1, 2$ from the parent.[13] The descendant does not leave bequests; her preferences are described by the strictly concave utility function $u^n(c^n, d^n, l^n)$ and her wage rate is w^n. I concentrate on the effect of a tax on the wealth transfer from the first to the second generation, given the LTC risk of the first generation. Therefore, I neglect

[12] In fact, the need for LTC may arise in several steps of increasing intensity. Uncertainty is removed completely only after the highest possible extent has occured.

[13] This reflects the fact that in reality inheritances are typically received later in life. Arrondel et al. (1997) report a mean age of 48 for France in 1984.

the LTC risk of the descendant generation.

The individual of the parent generation has dynastic preferences, that is, well-being of the descendant generation enters her utility function with the weight δ, $0 < \delta \leq 1$, which denotes the degree of altruism. Her decision problem reads:

$$\begin{aligned} \text{maximize} \quad & u(c, d, l) + \delta \sum_{i=0}^{2} \pi_i u^n(c^n, d_i^n, l^n) && (44) \\ \text{s.t.} \quad & (3),\ (4) \text{ and} \\ & c^n(1+\tau^n) + s^n \leq w^n l^n (1 - t^n), && (45) \\ & d_i^n(1+\tau^n) \leq s^n(1+r) + b_i(1+r), i = 0, 1, 2. && (46) \end{aligned}$$

In addition, the nonnegativity constraints c^n,s^n,$d_2^n \geq 0$ must hold. The $d_i^n, i = 0, 1, 2$ denote descendant retirement consumption, depending on the realization of the LTC risk of the parent. For the purpose of a comparison with the earlier model I assume here that the descendant generation can be taxed at specific rates τ^n, t^n.[14] Let c^{n*},s^{n*},d_i^{n*},l^{n*} denote optimum values of the descendant-generation variables, resulting as the solution of (44)-(46).

The descendant, who has to choose her labor supply and first-period consumption before the realization of the LTC risk of the parent, knows the possible values b_i and the corresponding probabilities $\pi_i, i = 0, 1, 2$. She solves the decision problem of maximizing

$$\sum_{i=0}^{2} \pi_i u^n(c^n, d_i^n, l^n), \tag{47}$$

subject to (45) and (46). One observes immediately that, given the appropriate b_i, the (time-consistent) solution is again c^{n*},s^{n*},d_i^{n*},l^{n*}.[15]

Now define parent utility from leaving bequests as $v^n(b_i) \equiv \delta u^n(c^{n*}, d^n(b_i), l^{n*})$ with $d^n(b_i) \equiv (s^{n*} + b_i)(1+r)/(1+\tau^n)$, that is, $d^n(b_i)$ is retirement consumption of the descendant, given an inheritance b_i and optimal values of the other variables. Then the objective

[14] See, however, footnote 16.

[15] One can obviously multiply the objective function of the descendant by δ without affecting the optimal values of the decision variables. Then the first-order conditions of this problem are the same as those of maximizing (44) subject to (3), (4), (45), (46) with respect to the descendant-generation variables c^n, s^n, d_i^n, l^n.

(44) of the parent can be written as $u(c,d,l)+\sum_{i=0}^{2}\pi_i v^n(b_i)$. From this formulation it is immediately visible that the joy-of-giving motive for leaving bequests, introduced in Section 2, can be interpreted as comprising a purely altruistic motive as well. As a consequence, the results derived in Sections 3 and 4 continue to hold in case of dynastic prefereces.[16]

However, a qualification has to be made concerning the objective function of the government. Employing a utilitarian framework, the government may consider social welfare as being the (weighted) sum of both generations' welfare and evaluate the welfare effect of a tax according to the objective function $W = u(c,d,l) + \delta\sum_{i=0}^{2}\pi_i u^n(c^n, d_i^n, l^n) + \beta\sum_{i=0}^{2}\pi_i u^n(c^n, d_i^n, l^n)$, with β, $0 \leq \beta \leq 1$ denoting a social discount factor of future generation's utility. The above result that the joy-of-giving motive can be interpreted as comprising altruistic preferences refers to the case that the government sets $\beta = 0$; then its objective is welfare of the first generation, which in a dynastic setting includes welfare of the descendant generation.

But with $\beta > 0$ the well-known issue of double counting arises: welfare of the second generation occurs twice, as part of parent utility and as a separate goal. As a consequence of this double counting there is a case for a subsidy on bequests, because they create a positive external effect, as mentioned in the Introduction (for a discussion see also Brunner and Pech 2012b). They provide twofold utility, for the donating parent as well as for the receiving descendant.

In case of double counting ($\beta > 0$) the above model with a joy-of-giving motive for leaving bequests does not apply any more; the reason being that social welfare differs from the representative parent's objective by the term $\beta\sum_{i=0}^{2}\pi_i u^n(c^n, d_i^n, l^n)$. That is, in order to determine the marginal *social* deadweight loss $-(\partial W/\partial j)/(\partial R/\partial j)$ of each tax $j = t, \tau, \sigma$, the influence on descendant utility (times β) has to be added. Whether this influence is positive or negative only depends on how the size of bequests b_i reacts to the respective tax.

[16] One can also compute the marginal deadweight losses m_{t^n} and m_{τ^n} of the second-generation taxes and finds that $m_{\tau^n} < m_{t^n}$, because τ^n raises a different amount from descendant consumption according to the bequests left by the parent. Whether the tax on descendant consumption is preferable to the bequest tax depends on the size of the relative variation of the respective tax base; see the similar discussion in 6.1. The plausible case is that the variation is larger for bequests (then $m_\sigma < m_{\tau^n}$) because τ^n also taxes first-period descendant consumption, which is independent of inheritances.

To determine this (see Appendix B for a detailed analysis) one observes that the bequest tax σ causes a substitution effect against bequests, while the consumption tax τ causes a substitution effect in favor of bequests. Indeed, for quasilinear preferences the reaction of bequests b_i in all three states can be shown to be positive for the consumption tax τ. Concerning the effect of σ, the reaction of b_0 can be shown to be negative, at least for additively separable utility functions, while the reaction of b_1 and b_2 is undetermined and may be positive. The effect on $b_i, i = 0, 1, 2$ is zero for the income tax t. As a consequence, compared to a situation without double counting, now m_σ is likely to become larger, m_t remains unaffected while m_τ becomes smaller. The overall finding is:

With dynastic preferences and $\beta = 0$ (no double counting) the results of Section 3 establishing the advantage of a bequest tax compared to an income and a consumption tax also apply from an intertemporal social-welfare perspective.

With dynastic preferences and $\beta > 0$ (double counting), and under the assumption of quasilinear utility, the results of Section 3 have to be modified if an intertemporal social-welfare perspective is adopted: Then the advantage of the bequest tax is reduced, more so in comparison with the consumption tax than with the income tax.

Obviously, the important ethical question in this context is whether double counting is appropriate. It may look appealing at first glance, given the utilitarian framework. One should, however, be clear about its consequences: it calls for a subsidy (or a tax on consumption) to increase bequests of the parent above the level she finds optimal herself without government intervention. Thus, it effectively implies redistribution from the parent to the descendant, because the distortion makes the parent generation worse off. Importantly, the increase of the (altruistic) bequests, which represent a pure transfer of resources, does not lead to a net welfare gain (no potential Pareto improvement, see also Milgrom 1993), which makes the situation different from a subsidy of some consumption good producing a positive external effect. For this reason several authors have argued against double counting (see Hammond 1987 and Harsanyi 1995).

7 Concluding remarks

In this paper I have shown that a tax on bequests has a distinct advantage over an income or a consumption tax, because it provides insurance against the reduction of bequests through the need for LTC. By making the tax liability contingent on the state of LTC need, it allows a differentiated treatment of individuals according to the realization of uncertain LTC expenditures, which clearly is not possible with an income tax or a consumption tax.

This result holds for a proportional tax on bequests. Given that the extent of LTC need (the realization of x_i) is assumed to be observable, the government can choose a form of estate taxation where the tax rate itself depends on the amount of LTC expenditures the individual actually has to bear. This increases the number of available instruments and can, thus, only lead to higher welfare, compared to the imposition of a single tax rate. It turned out that the tax imposed in a situation of lowest LTC expenditures (largest bequests) is the unambiguosly best instrument, while the tax imposed in case of highest expenditures (lowest bequests) causes a larger marginal deadweight loss than the income tax and the consumption tax.

While for the presentation of the main idea I formulated bequests as being motivated by joy-of-giving, I have shown that the results hold in the same way if dynastic preferences are assumed. A difference occurs, however, if welfare of the descendant generation is counted twice in the social objective, because this in fact calls for a subsidy of bequests and counteracts, from an intertemporal social-welfare perspective, the positive insurance effect of the bequest tax.

Obviously, in the model of this paper the advantage of a bequest tax hinges on the assumption that the individual indeed wants to leave bequests. If she has no bequest motive, that is, if $v(b) = 0$ in the model, then she may only leave unintended bequests for which the logic developed in this paper does not apply. However, one could think of a model where an individual without a bequest motive saves, when young, to prepare for uncertain LTC expenditures. Of course, the best strategy would be to buy private LTC insurance. If she does not follow this strategy (for instance, because of overpriced contract offers due to adverse selection and other problems), she may save too much and leave unintended bequests, if LTC expenditures turn out to be low. Then a tax on bequests (and a lower tax on income or consumption) works as a substitute for insurance. This reasoning is clearly related to the idea mentioned in the Introduction that a bequest tax provides insurance against *unintended* bequests, in case of premature death and imperfect annuitization of wealth.

Appendix A

Let preferences be of the form $u(c,d,l) = \widetilde{u}(c,d) - u^l(l)$, with $\widetilde{u}$ linear-homogeneous and strictly concave, u^l strictly convex. To see that for this type of preferences the cross price effects of σ are zero, while the own and cross price effects of t and τ are non-positive, remember that in case of linear homogeneity the condition $\widetilde{u}_c/\widetilde{u}_d = 1+r$ determines a fixed ratio between the amounts consumed of c and d which is independent of any tax. Moreover, as a property of linear-homogeneous functions, marginal utility is $\widetilde{u}_c$ is constant along this ray through the origin. By (6) this determines $\lambda = \widetilde{u}_c/(1+\tau)$. Thus, l is uniquely determined by (8), which now reads as $u^{l\prime} = w(1-t)/(1+\tau)$ and depends (negatively) on the income tax t and on the consumption tax τ, but not on the bequest tax σ (neither does λ). b_0 as well as b_1, b_2 result from (9) and (4); they are negatively influenced by σ as can be found by implicit differentiation of (9), after substitution of (4), $i = 1, 2$:

$$\begin{aligned} \frac{\partial b_0}{\partial \sigma} &= -[\partial(\pi_0 v'(b_0) + \sum_{i=1}^{2} \pi_i v'(b_0 - \frac{x_i - a_i}{1+\sigma}) - \frac{\lambda(1+\sigma)}{1+r})\partial\sigma]/\sum_{i=0}^{2} \pi_i v''(b_i), \qquad (48) \\ &= [-\sum_{i=1}^{2} \pi_i v''(b_i)\frac{x_i - a_i}{(1+\sigma)^2} + \frac{\lambda}{1+r}]/\sum_{i=0}^{2} \pi_i v''(b_i). \qquad (49) \end{aligned}$$

The denominator on the RHS is negative, while the numerator is positive, hence b_0 as well as b_1, b_2 decrease with the tax rate σ. Finally, the exact amounts of c and d follow from the budget constraint (3), they obviously decrease with an increase of the income or the consumption tax.

A further example of suitable preferences is described by the utility function $u(c,d,l) = c + u^d(d) - u^l(l)$ with u^d strictly concave and u^l strictly convex. Here u_c is equal to 1, and λ is computed from (6) as $1/(1+\tau)$. d is determined from (7), $u^{d\prime} = 1/(1+r)$, and is independent of all tax rates. l follows from (8), $u^{l\prime} = w(1-t)/(1+\tau)$; clearly l depends negatively on t and τ. b_0 follows from (9) and (4), $i = 1, 2$. Substituting $\lambda = 1/(1+\tau)$ and differentiating (9) implicity leads one to the result that again b_0 as well as b_1, b_2 decrease with the tax rate σ. Moreover, c is residually determined from the budget constraint and depends negatively on the income and the consumption tax.

Appendix B

In order to analyze the case of double counting, I write social welfare as $W = U(t,\tau,\sigma) + \beta\overline{u}^n(b_0(t,\tau,\sigma), b_1(t,\tau,\sigma), b_2(t,\tau,\sigma))$, where U is indirect utility of the parent, obtained as the solution of (44), (3), (4), (45), (46). $\overline{u}^n(\cdot)$ is the optimum value of the descendant's welfare (47) subject to (45), (46), depending on potential inheritances b_i, which result from the parent's decision. The marginal loss of social welfare due to an increase of some tax rate $j = t, \tau, \sigma$ is now given by $\partial W/\partial j = \partial U/\partial j + \beta \sum_{k=0}^{2} (\partial\overline{u}^n/\partial b_k)(\partial b_k/\partial j)$. Then, proceeding as in Section 3 and comparing the marginal social deadweight loss $m_j^s \equiv -(\partial W/\partial j)/(\partial R/\partial j)$ associated with the various taxes, now involves the additional term $-\beta \sum_{k=0}^{2} (\partial\overline{u}^d/\partial b_k)(\partial b_k/\partial j)$ in the numerator. Therefore, as the $\partial\overline{u}^n/\partial b_k$ are positive, the consequence of double counting depends on the value of $\partial b_k/\partial j$, that is, how the tax affects bequests.

To determine these derivatives, the decision problem of the parent, (44), (3), (4), (45), (46) is written as maximization of $u(c,d,l) + \widetilde{v}(b_o,\sigma)$, subject to (3), where $\widetilde{v}(b_o,\sigma)$ is defined as the optimal value of

$$\begin{aligned} \underset{c^n, d_0^n, l^n}{\text{maximize}} \quad & \delta(\pi_0 u^n(c^n, d_0^n, l^n) + \pi_1 u^n(c^n, d_0^n - \frac{x_1 - a_1}{1+\sigma}\frac{1+r}{1+\tau^n}, l^n) + \qquad (50) \\ & + \pi_2 u^n(c^n, d_0^n - \frac{x_2 - a_2}{1+\sigma}\frac{1+r}{1+\tau^n}, l^n), \\ \text{s.t.} \quad & (c^n + \frac{d_0^n}{1+r})(1+\tau^n) \le w^n l^n (1-t^n) + b_0 \qquad (51) \end{aligned}$$

This formulation is obtained by eliminating s^n from (45) and (46) for $i = 0$, and expressing d_1^n, d_2^n, b_1, b_2 in terms of d_0^n and b_0 by using (4) and (46) for $i = 1, 2$. Note that $\widetilde{v}$ is strictly concave in b_0, due to the strict concavity of u^n and convexity of the feasible set.

The first-order conditions for the parent's problem are given by (6), (7), (8) and

$$\frac{\partial \widetilde{v}}{\partial b_0} - \lambda \frac{1+\sigma}{1+r} = 0. \tag{52}$$

Next assuming that parent preferences are of the form $u(c,d,l) = \widetilde{u}(c,d) - u^l(l)$, with $\widetilde{u}$ linear-homogeneous and concave, u^l strictly convex, the same reasoning as in Appendix A shows that $\lambda = \widetilde{u}_c/(1+\tau)$ is decreasing in τ, but independent of t and σ. Hence from implicit differentiation of (52) and $\partial^2 \widetilde{v}/\partial b_0^2 < 0$, it follows that b_0 is increasing in τ and independent of t ($\widetilde{u}_c$ is constant, independent of any tax, $\widetilde{v}$ is independent of τ and t). Further, (4) shows that the influence of τ and t on b_1 and b_2 is the same as that on b_0.

Concerning the influence of σ, implicit differentiation of (52) gives $\partial b_0/\partial\sigma = -(\partial^2\widetilde{v}/\partial b_0 \partial\sigma - \lambda/(1+r))/(\partial^2\widetilde{v}/\partial b_0^2)$, which is negative provided that $\partial^2\widetilde{v}/\partial b_0\partial\sigma < \lambda/(1+r)$. To check this, consider the system of equations consisting of the first-order conditions for (50) and (51) with respect to c^n, d_0^n, l^n, and of the budget constraint (51):

$$\begin{aligned}
\delta \sum_{i=0}^{2} \pi_i u_c^n(\cdot) - \nu(1+\tau^n) &= 0,\\
\delta \sum_{i=0}^{2} \pi_i u_{d_i}^n(\cdot) - \nu \frac{1+\tau^n}{1+r} &= 0,\\
\delta \sum_{i=0}^{2} \pi_i u_l^n(\cdot) + \nu(1-t^n) &= 0,\\
(c^n + \frac{d_0^n}{1+r})(1+\tau^n) - w^n l^n (1-t^n) &= b_0.
\end{aligned}$$

Here ν denotes the Lagrangean variable corresponging to (51), which is equal to $\partial\widetilde{v}/\partial b_0$ by the Envelope Theorem. Implicit differentiation with respect to σ gives

$$\begin{pmatrix} \frac{\partial c^n}{\partial\sigma} \\ \frac{\partial d_0^n}{\partial\sigma} \\ \frac{\partial l^n}{\partial\sigma} \\ \frac{\partial \nu}{\partial\sigma} \end{pmatrix} = - \begin{pmatrix} \delta\sum_{i=0}^{2}\pi_i u_{cc}^n & \delta\sum_{i=0}^{2}\pi_i u_{cd_i}^n & \delta\sum_{i=0}^{2}\pi_i u_{cl}^n & -(1+\tau^n) \\ \delta\sum_{i=0}^{2}\pi_i u_{cd_i}^n & \delta\sum_{i=0}^{2}\pi_i u_{d_id_i}^n & \delta\sum_{i=0}^{2}\pi_i u_{d_il}^n & -\frac{1+\tau^n}{1+r} \\ \delta\sum_{i=0}^{2}\pi_i u_{cl}^n & \delta\sum_{i=0}^{2}\pi_i u_{d_il}^n & \delta\sum_{i=0}^{2}\pi_i u_{ll}^n & w^n(1-\tau^n) \\ -(1+\tau^n) & -(\frac{1+\tau^n}{1+r}) & w^n(1-\tau^n) & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} \sum_{i=1}^{2}\pi_i u_{cd_i}^n \frac{x_i-a_i}{(1+\sigma)^2}\frac{1+r}{1+\tau^n} \\ \sum_{i=1}^{2}\pi_i u_{d_id_i}^n \frac{x_i-a_i}{(1+\sigma)^2}\frac{1+r}{1+\tau^n} \\ \sum_{i=1}^{2}\pi_i u_{ld_i}^n \frac{x_i-a_i}{(1+\sigma)^2}\frac{1+r}{1+\tau^n} \\ 0 \end{pmatrix}$$

In general, the sign of $\partial\nu/\partial\sigma$ is undetermined. However, in case of additive separability between d^n and (c^n, l^n), that is $u^n_{cd_i} = u^n_{ld_i} = 0$, straightforward computation shows that $\partial\nu/\partial\sigma = -[(1+\tau^n)\delta^2(u^n_{cc}u^n_{ll} - u^{n2}_{cl})\sum\limits_{i=1}^{2} u^n_{d_id_i}(x_i - a_i)(1+r)/((1+\sigma)^2(1+\tau^n))]/\det(X)$, where $\det(X)$ is the determinant of the above bordered Hessian, which is negative due to the strict concavity of u^n.[17] The latter also implies $u^n_{cc}u^n_{ll} - u^{n2}_{cl} > 0, u^n_{d_id_i} < 0$, hence we get $\partial\nu/\partial b_0 < 0$.

Thus, $\partial^2\widetilde{v}/\partial b_0\partial\sigma < 0$ and $\partial b_0/\partial\sigma < 0$ hold for additive utility function, and one can conclude that $\partial b_0/\partial\sigma < 0$ even holds for a broader class of utility functions, as in fact only $\partial^2\widetilde{v}/\partial b_0\partial\sigma < \lambda/(1+r)$ is required. On the other hand, it follows from (4) that $\partial b_1/\partial\sigma$ and $\partial b_2/\partial\sigma$ are both larger than $\partial b_0/\partial\sigma$, because $\partial(-(x_i - a_i)/(1+\sigma))/\partial\sigma > 0$, hence they may be positve.

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[17] See, e. g. Sydsaeter et. al 2008, p. 75. Note from the first-order conditions that the last row and column of the bordered Hessian are equal to the partial derivatives of the objective times ν. The factor ν does not change the sign of the deminant. Moreover, observe that in case of additive separability the marginal utilities of c^n and l^n are the same for all realizations d^n_i, hence $\sum\limits_{i=0}^{2}\pi_i u^n_{cc} = u^n_{cc}$, and the same for u^n_{cl} and u^n_{ll}.

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