The federal funds market, excess reserves, and unconventional monetary policy

by

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Abstract

Following the bankruptcy of Lehman Brothers, interbank borrowing and lending dropped, whereas reserve holdings of depository institutions skyrocketed, as the Fed injected liquidity into the U.S. banking sector. This paper introduces bank liquidity risk and limited market participation into a real business cycle model with ex ante identical financial intermediaries and shows, in an analytically tractable way, how interbank trade and excess reserves emerge in general equilibrium. Investigating the role of the federal funds market and unconventional monetary policy for the propagation of aggregate real and financial shocks, I find that federal funds market participation is irrelevant in response to standard supply and demand shocks, whereas it matters for “uncertainty shocks”, i.e. mean-preserving spreads in the cross-section of liquidity risk. Liquidity injections by the central bank can absorb the effects of financial shocks on the real economy, although excess reserves might increase and federal funds might be crowded out, as a side effect.

Keywords: Excess reserves; Federal funds market; Financial frictions; Liquidity risk; Unconventional monetary policy

JEL Classification: C61; E32; E51; E52

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1 Introduction

Following the bankruptcy of Lehman Brothers, the Federal Reserve injected substantial amounts of liquidity into the U.S. banking sector in order to contain tensions in the federal funds market. As a result, the reserve holdings of depository institutions skyrocketed. Pundits have taken this as a signal of ineffectiveness of the Fed’s liquidity facilities in promoting the supply of credit to the economy (compare Keister and McAndrews, 2009). Figure 1 illustrates that excess reserves used to play only a negligible role on the balance sheets of financial intermediaries throughout the post-war period, averaging less than 0.05% of total deposits before September 2008. After Lehman, excess reserves increased from virtually zero to almost 20%, whereas interbank lending, i.e. federal funds and reverse repurchase agreements of U.S. commercial banks with other banks, dropped from 5.5% to less than 1.5% as a fraction of total deposits.

Identifying federal funds transactions based on Fedwire data, Ashcraft et al. (2011) document a dramatic increase in bank liquidity risk in August 2007, due to looming intraday payments for asset-backed commercial paper liquidity lines, whereas concerns about increased counterparty risk in interbank transactions were largely irrelevant until October 2008. Likewise, Afonso and Lagos (2012) show that the average number of daily trades and counterparties dropped from above 860 to about 240 and from 4.5 to 3, respectively, whereas average loan size doubled during the financial crisis of 2007–2009. Hence, the drop in the total volume of federal funds since 2008 was driven by a reduction in trade frequency rather than in trade size.

The aim of this paper is to show, in an analytically tractable way, how interbank lending and excess reserves can emerge in general equilibrium. For this purpose, I introduce liquidity risk in the spirit of Rochet and Tirole (1996) and Holmström and Tirole (1998) and limited federal funds market participation in an otherwise frictionless real business cycle (RBC) model with financial intermediation. The model can thus capture the prevalence of interbank credit before as well as the rise in reserves during the “Great Recession” and is used to investigate the role of the federal funds market and unconventional monetary policy for the propagation of aggregate real and financial shocks.

In the model, interbank borrowing and lending arises from idiosyncratic uncertainty of ex ante identical financial intermediaries. Building on Dib (2010), banks extend commercial loans to

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1Aggregate excess reserves are defined as total minus required reserves of all depository institutions. All data are from the Board of Governors of the Federal Reserve and seasonally adjusted.

2Throughout the paper, I use the terms “federal funds” and “interbank loans” interchangeably, when referring to unsecured short-term credit between financial intermediaries.
goods-producing firms according to a Leontief technology, i.e., they must simultaneously fulfill a non-negativity constraint on liquidity and a minimum capital requirement, as in the Basel accords. Deposits are stochastic at the bank level and realize only after a bank has chosen its desired volume of loans and bank capital. Each individual bank is thus subject to liquidity risk and an occasionally binding liquidity constraint. Institutions with low (high) deposit realizations relative to their bank capital stock can engage in interbank borrowing (lending). Whether a given bank participates in the federal funds market is exogenous, however. I abstract from pro-cyclical balance sheet constraints in financial intermediation. Hence, my model represents a complement rather than a substitute for the well-known “financial accelerator”.

Extending the approach in Wen (2011), I aggregate the occasionally binding liquidity constraints of individual banks by assuming a suitable distribution function of their idiosyncratic deposit realizations. The model can thus capture microeconomic uncertainty at the bank level even in a loglinear approximation around the steady state.

With full participation in the federal funds market, interbank borrowing and lending provides perfect insurance against liquidity risk. Accordingly, the banking sector is “self-sufficient” in the sense of Holmström and Tirole (1998), attaining an optimal equilibrium allocation of liquidity. Limited participation induces banks to hold excess reserves, raises the interest rate spread in financial intermediation, and impairs thus steady-state real economic activity through a credit cost channel (compare Kiyotaki and Moore, 2008).

Given a symmetric distribution function of deposit realizations and an exogenous, in particular acyclical, participation in the federal funds market, the latter is irrelevant for the propagation of conventional supply and demand shocks. On the contrary, the federal funds market is crucial for attenuating “uncertainty shocks”, i.e. changes in the variance of banks’ deposit realizations. An exogenous decrease in participation triggers a simultaneous increase in excess reserves and an economic recession.

Unconventional monetary policy, modeled here as a liquidity injection by the central bank, can reduce banks’ liquidity risk and stabilize real economic activity in response to financial shocks. Depending on its implementation, however, it might lead to an even more pronounced increase in excess reserves and a crowding out of interbank credit.

My work is most closely related to Gertler and Kiyotaki (2011). In their model, a continuum of financial intermediaries is located on a continuum of islands. Each period, a fraction of firms is randomly assigned the possibility to invest (compare also Kiyotaki and Moore, 2008). Financial intermediaries can only lend to firms on the same island but to intermediaries on all islands. Asymmetric information implies endogenous borrowing constraints both in the “retail” market...
for deposits and the “wholesale” market for interbank credit.

Assuming a normal distribution of the stochastic deposit realizations (see, e.g., Frost, 1971), idiosyncratic uncertainty in my model is, at the same time, less complex and more general than in Gertler and Kiyotaki (2011). (i) Aggregate interbank credit, excess reserves, and commercial loans have an analytical solution. (ii) The functional form assumption facilitates uncertainty shocks, i.e. mean-preserving spreads in the distribution of liquidity shocks (see also Williamson, 1987). (iii) Given the focus on bank reserves and the federal funds market, in the present paper, “unconventional monetary policy” refers to the Fed’s liquidity facilities rather than the direct intermediation of credit by the central bank, as in Gertler and Karadi (2011).

The model in this paper bridges a gap in the literature on business cycles and financial frictions. Excess reserves cannot be captured in a deterministic framework, where profit-maximizing banks hold zero reserves except for legal requirements. When subject to uncertain in- and outflows of liquidity, however, it is rational for individual banks to insure against negative liquidity shocks by holding precautionary reserves. In the presence of protection against liquidity risk, in turn, reserves become irrelevant (compare Poole, 1968). Numerous microeconomic contributions show that a frictionless interbank market provides perfect insurance by pooling independent liquidity risks (see, e.g., Bhattacharya and Fulghieri, 1994; Freixas et al., 2000).

Until recently, financial frictions in macroeconomics were mostly confined to the limited access of firms to external finance, arising from asymmetric information between financial intermediaries and ultimate borrowers (see, e.g., Williamson, 1987; Bernanke and Gertler, 1989).\(^3\) However, the financial crisis of 2007–2009 originated from frictions within the banking sector, before spreading to the real economy (compare Brunnermeier, 2009, and Woodford, 2010). Nevertheless, only a small strand of the macroeconomic literature considers the effects of changes in credit conditions that are unrelated to borrower characteristics, as in Cook (1999), Gertler and Kiyotaki (2011), and Gertler and Karadi (2011).

Few macroeconomic models are able to generate excess reserves. If so, reserves are assumed to be an input into a neoclassical production function for transaction services (see, e.g., Chari et al., 1995; Christiano et al., 2010).\(^4\) Interbank borrowing and lending merely occurs between banks that differ in their structural parameters, as in Dib (2010) and Gerali et al. (2010). While some authors refer to these transactions as “federal funds”, financial intermediation is effectively split

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\(^3\) An extensive literature documents the quantitative importance of the financial accelerator in accounting for business cycle fluctuations, both theoretically (see, e.g., Carlstrom and Fuerst, 1997; Bernanke et al., 1999) and empirically (see, e.g., Christensen and Dib, 2008).

\(^4\) Chari et al. (1995) acknowledge that, dropping the production function for demand deposits, the nonnegativity constraint on excess reserves would hold with strict equality. In Christiano et al. (2010), reserves are explicitly included into the production of liquidity services “as a reduced form way to capture the precautionary motive of a bank concerned about […] unexpected withdrawals”.
up into the collection of deposits and the provision of loans. Different rates of time preference of depository and lending institutions thus facilitate a one-way stream of interbank credit from patient to impatient, i.e. “between” ex ante heterogeneous financial intermediaries.

The rest of the paper is organized as follows. Section 2 describes the model’s timing and agents, derives their individual decision rules and the aggregate equilibrium conditions. The calibration of parameters and benchmark steady-state values are presented in section 3. Sections 4 and 5 illustrate the dynamic implications of federal funds trade and unconventional monetary policy, respectively. Finally, I relate the model’s theoretical predictions to the response of the Fed to the recent financial crisis and discuss the implications for monetary policy. Section 6 concludes.

2 The Model

The model is set up in infinite discrete time and comprises five types of economic agents: A representative worker household, a representative banker household, a representative capital goods producer, a representative entrepreneur and final goods producer, and a decentralized banking sector.

The worker household enters each period with a predetermined level of bank deposits, supplies homogeneous labor services and receives shareholder dividends from the final goods-producing firm. Household income is either consumed or saved for subsequent periods in terms of bank deposits, which yield a riskless real rate of return.

The banker household does not work, derives utility from consumption, and accumulates bank capital, which is then provided to financial intermediaries. As in Dib (2010), bank capital or equity pays a state-contingent real rate of return (dividend). I abstract from agency problems between households and financial intermediaries.

The banking sector provides financial intermediation between lenders and borrowers by converting household deposits into loans to entrepreneurs. Ultimate savers cannot lend directly to ultimate borrowers at a single common interest rate – a feature of many pre-crisis models criticized by Woodford (2010). Banks must satisfy a stylized minimum capital requirement mimicking Basel II in order to lend to non-financial corporations. When choosing an optimal level of bank capital, each bank knows the value of aggregate household deposits but does not know its idiosyncratic deposit realization. Once the bank-specific liquidity shocks have occurred, financial institutions might borrow or lend in an interbank market.

5The liquidity shocks could equivalently be modeled in terms of stochastic deposit in- and outflows. Since one bank’s outflow is usually another bank’s inflow (compare Kaufman and Lombda, 1980), the corresponding probability distribution would reasonably be mean zero. More generally, the uncertainty in this model should be thought of as short-term funding uncertainty. Another possibility would be to introduce uncertainty about banks’ investment opportunities, i.e. on the asset side, as in Gertler and Karadi (2011).
Entrepreneurs cannot accumulate own net worth and rely on external finance in order to acquire the capital stock in advance of production. When the physical capital stock becomes productive in the subsequent period, the representative entrepreneur hires labor services from the worker household. After production, the depreciated capital is sold back to capital goods producers and the bank loan is repaid. Any profits or losses are distributed to the firm’s shareholders – the worker household. The representative capital goods producer recycles the predetermined depreciated capital stock and invests, before selling the new productive capital to entrepreneurs.

2.1 The Banking Sector

A unit-mass continuum of ex-ante identical banks is indexed on the interval \([0, 1]\). Banks enter each period with zero assets and zero liabilities. Similar to Dib (2010), banks use a common Leontief production function for providing loans. On the one hand, each bank \(i\) must satisfy a stylized capital requirement by holding bank capital \(z_t(i)\). On the other hand, lending to entrepreneurs is bounded above by the bank’s disposable liquidity:

\[
l_t(i) = \min \{\bar{\kappa}z_t(i), d_t(i) + b_t(i) + z_t(i)\},
\]

where \(\bar{\kappa}\) denotes the maximum admissible bank loans per unit of bank capital, i.e. the inverse of the minimum bank capital ratio specified in the Basel II accord. Each period, bank \(i\) is subject to a stochastic deposit realization, \(d_t(i)\), that is i.i.d. across banks and through time. \(b_t(i)\) denotes federal funds borrowing or lending of bank \(i\).

The strict complementarity between the arguments in the Leontief production function should not be taken literally, but serves as a modeling device to approximate the timing of liquidity shocks in a three-period agency model (see, e.g., Rochet and Tirole, 1996). Suppose, e.g., that the market for bank capital opens before the market for deposits, or that intermediaries are subject to stochastic deposit in- and outflows throughout the period. As a consequence, banks must commit to one input in (1) without knowing the quantity of the other input.

After \(z_t(i)\) has been chosen, the idiosyncratic deposit shocks occur and each bank learns its \(d_t(i)\). Following a high (low) deposit realization relative to its stock of capital, bank \(i\) can lend (borrow) in an interbank market, which facilitates the reallocation of liquidity between financial intermediaries – a crucial characteristic of the U.S. federal funds market (see, e.g., Keister and McAndrews, 2009; Afonso and Lagos, 2012). In the model, \(b_t(i)\) represents uncollateralized short-term borrowing (if \(b_t(i) > 0\)) and lending (if \(b_t(i) < 0\)) between financial intermediaries.

For these reasons, the terms “interbank market” and “federal funds market” are used interchangeably in this paper. The maximum disposable funds bank \(i\) can lend to entrepreneurs
thus amount to $d_t(i) + b_t(i) + z_t(i)$.

At this point, I deliberately omit alternative ways of raising liquidity, after individual uncertainty is resolved, such as the discount window, for example. Saunders and Urich (1988), Peristiani (1998), and Furfine (2001) provide empirical evidence that banks are reluctant to borrow from the discount window in “normal” times, fearing that the Federal Reserve and other financial institutions might interpret this as a signal of poor liquidity management or financial distress. Ennis and Weinberg (2013) study a theoretical model of the stigma attached to discount window borrowing.

Suppose that bank $i$ expects a volume $E_t d_t(i)$ of deposits in period $t$, whereas its actual realization, $d_t(i)$, deviates from this expectation with known exogenous variance, $\sigma_t^2$. The uncertainty in my model is thus purely idiosyncratic. In contrast to Holmström and Tirole (1998), I omit the case, where all banks experience the same liquidity shock. Based on the incomplete information about its available liquidity, bank $i$ chooses an optimal level of $z_t(i)$ in order to maximize expected profits in period $t + 1$,

$$E_{t} \pi_{t+1}(i) := (1 + r^L_t) E_t l_t(i) + (1 + r^R_t) E_t r_t(i) - (1 + r^D_t) E_t d_t(i) - (1 + r^B_t) E_t b_t(i) - (1 + E_{t+1}^Z_t) z_t(i), \quad (2)$$

subject to the balance sheet identity,

$$l_t(i) + r_t(i) = d_t(i) + b_t(i) + z_t(i), \quad (3)$$

and a nonnegativity constraint on its liquid reserves,

$$r_t(i) \geq 0. \quad (4)$$

In (2), $r^L_t$, $r^R_t$, $r^D_t$, and $r^B_t$ denote the real rates of return on bank loans, liquid reserves, bank deposits, and federal funds, respectively, while $r^Z_t$ denotes the state-contingent return on equity. As in Ashcraft et al. (2011), banks are not required to hold reserves. In practice, required reserves correspond to a constant fraction of deposits and contributed very little to the recent increase in total reserve balances.

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6For concreteness, I assume that the probability distribution of $d_t(i)$ is sufficiently described by its first and second moment.

7In the data, there is no evidence of an aggregate run on banks by depositors. For the same reason, I do not introduce an agency problem between financial intermediaries and depositors, as in Gertler and Karadi (2011). Due to the fact that the Leontief technology allows for “partial liquidation” of investment projects at the individual bank level, the consequences of aggregate uncertainty would be less dramatic than in Rochet and Tirole (1996) and Holmström and Tirole (1998), where negative excess liquidity leads to a termination of projects and the productive sector’s ex post value goes to zero.
Substituting for $E_t r_t(i)$ from (3), expected profits of bank $i$ simplify to

$$E_t \pi^b_{t+1}(i) := (r^L_t - r^R_t)E_t l_t(i) - (r^D_t - r^R_t)E_t d_t - (r^B_t - r^R_t)E_t b_t(i) - (r^Z_t - r^R_t)z_t(i).$$

Note that I abstract from the issue of default in any of the banks’ assets or liabilities. It is straightforward to account for the failure of bank loans to entrepreneurs by introducing a productivity shock that represents the fraction of non-defaulting loans in equation (1). An exogenous reduction in its cross-sectional expectation would then induce banks to raise $r^L_t$, whereas the conclusions in the present paper would not be affected.

### 2.2 The Household Sector

#### 2.2.1 Workers

The infinitely-lived representative worker derives positive utility from consumption, $C^w_t$, of the final good and negative utility from hours spent working in goods production, $N_t$. Labor services are rewarded with a real wage rate of $w_t$. Consumption can be transferred across periods by means of bank deposits, $D_t$, which yield a riskless rate of return $r^D_t$. Note that $D_t \equiv \int_0^1 d_t(i)di$ and that $D_t$ is a decision variable of the representative worker household, whereas the fraction of deposits transferred to bank $i$ is stochastic.

The worker chooses $\{C^w_t, N_t, D_t\}$ to maximize expected discounted lifetime utility,

$$E_t \sum_{\nu=0}^{\infty} (\beta_w)^\nu \left\{ \zeta_{t+\nu} \ln C^w_{t+\nu} - a \frac{N_{t+\nu}^{1+\gamma}}{1+\gamma} \right\},$$

subject to the budget constraint in period $t$,

$$C_t^w + D_t \leq w_t N_t + (1 + r^D_t)D_{t-1} + \pi^e_t,$$

where $\pi^e_t$ are dividend payments received from goods producers. $\beta_w$ denotes the worker’s subjective discount factor, $\gamma$ the inverse Frisch elasticity of labor supply, and $a$ a scaling parameter. $\ln(\zeta_t) = \rho \ln(\zeta_{t-1}) + \epsilon^\zeta_t$ is an autocorrelated consumption preference shock.

The corresponding first order conditions (FOCs) with respect to $C^w_t$, $N_t$, and $D_t$ are:

$$C^w_t : \quad \lambda^w_t C^w_t = \zeta_t,$$

$$N_t : \quad \lambda^w_t w_t = a (N_t)^\gamma,$$

$$D_t : \quad \lambda^w_t = \beta_w E_t \lambda_{t+1}^w (1 + r^D_t),$$

where $\lambda^w_t$ is the Lagrange multiplier of the budget constraint. Equations (6), (7), (8), and (9) represent the worker household’s equilibrium conditions.
2.2.2 Bankers

The infinitely-lived representative banker has zero labor endowment, derives utility from consumption of the final good, \( C^b_t \), and accumulates bank capital, \( Z_t \). Household income arises from the state-contingent return (dividend) \( r_t^Z \) on bank capital as well as from any residual profits in the banking sector. Note that \( Z_t \equiv \int_0^1 z_t(i)di \) and that the fraction of bank capital received by bank \( i \), \( z_t(i) \), is assumed to be non-stochastic in order to keep financial intermediation tractable. The banker chooses \( \{C^b_t, Z_t\} \) to maximize expected discounted lifetime utility,

\[
E_t \sum_{\nu=0}^{\infty} (\beta_b)^\nu \left( \zeta_{t+\nu} \ln C^b_{t+\nu} \right),
\]

subject to the budget constraint in period \( t \),

\[
C^b_t + Z_t \leq (1 + r_t^Z)Z_{t-1} + \pi^b_t,
\]

where \( \pi^b_t \) denotes aggregate banking sector profits, and \( \beta_b \) is the banker’s subjective discount factor. The remaining parameters in the utility function and the consumption preference shock are identical to those in (5). In the absence of inflation, the real stock of bank capital does not depreciate over time, when used by banks.

The banker household’s FOCs with respect to \( C^b_t \) and \( Z_t \) are:

\[
C^b_t : \quad \lambda^b_t C^b_t = \zeta_t, \tag{12}
\]

\[
Z_t : \quad \lambda^b_t = \beta_b E_t \left[ \lambda^b_{t+1} (1 + r_{t+1}^Z) \right], \tag{13}
\]

where \( \lambda^b_t \) is the Lagrange multiplier of the corresponding budget constraint. Equations (11), (12), and (13) represent the banker’s equilibrium conditions.

The main motivation for modeling a separate banker household is to have some leeway in the calibration of interest rates. Comparing equations (9) and (13), it is obvious that the simultaneous accumulation of deposits and bank capital by a single worker-banker household implies identical interest rates on \( D_t \) and \( Z_t \) in equilibrium, up to first order. Setting \( \beta_b < \beta_w \) facilitates a quantitatively relevant spread between the interest rates on deposits and bank loans.\(^8\) Alternatively, I could assume that a strictly positive share of loans defaults in the aggregate and that banks demand a risk premium on \( l_t(i) \).

\(^8\)Note that my qualitative results do not depend on the assumption of a less patient banker household and would persist for \( \beta_b = \beta_w \). Alternatively, one could assume that financial intermediaries can divert a fraction of bank capital, as in Dib (2010), effectively introducing default on bank capital.
2.3 The Production Sector

2.3.1 Capital Goods

After production in period \( t \), a representative capital goods producer buys the depreciated capital stock, \((1 - \delta)K_{t-1}\), and chooses optimal investment activity, \( I_t \), to maximize the present value of expected future discounted profits in terms of the worker’s utility,

\[
E_t \sum_{\nu=0}^{\infty} (\beta_w)^\nu \lambda_{t+\nu} \{ q_{t+\nu} [(1 - \delta)K_{t-1+\nu} + I_{t+\nu}] - I_{t+\nu} - q_{t+\nu}(1 - \delta)K_{t-1+\nu} \}
\]

subject to the equation of motion of the aggregate capital stock:

\[
K_t = (1 - \delta)K_{t-1} + I_t. \tag{15}
\]

The equality in (14) results from the assumption that the capital goods producer acquires the depreciated and sells the new capital stock at the same real price of capital, \( q_{t+\nu} \). As a consequence, she never realizes any capital gains or losses. Perfect competition implies zero profits in the capital goods-producing sector in each state of the economy.

The FOC with respect to investment and the equation of motion of the aggregate capital stock determine the capital market equilibrium. In the absence of investment adjustment costs, \( q_t = 1, \forall t \).

2.3.2 Final Goods

Entrepreneurs combine capital and labor services to produce a homogeneous final output good according to the following Cobb-Douglas production function:

\[
Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{16}
\]

where \( \ln(A_t) = \rho_A \ln(A_{t-1}) + \xi_t^A \) represents a transitory total factor productivity (TFP) shock. Note that the capital stock bought at the end of a period becomes productive in the subsequent period, only. E.g., \( K_{t-1} \) is predetermined in period \( t \) production.

The representative entrepreneur cannot accumulate own net worth and relies on external finance in order to acquire the physical capital stock in advance of production. Similar to Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), there is no agency problem between the financial intermediary and the firm. Accordingly, the real amount borrowed, \( L_{t-1} \), equals the quantity of productive capital in period \( t - 1 \):

\[
L_{t-1} = K_{t-1}. \tag{17}
\]
The entrepreneur chooses $K_{t-1}$ in period $t-1$ and $N_t$ in period $t$ in order to maximize the present value of expected future discounted profits in terms of the worker household’s utility,

$$E_t \sum_{\nu=0}^{\infty} (\beta^\nu \lambda^\nu \{Y_{t+\nu} + (1 - \delta)K_{t-1+\nu} - w_{t+\nu}N_{t+\nu} - (1 + r_{t+\nu})L_{t-1+\nu}\},$$

where $r_t^L$ is the real interest rate on a bank loan obtained at the end of period $t$.

Profit maximization requires that, up to first order, the expected real marginal return on the capital stock used in production in $t+1$ equals the real marginal cost per unit of $K_t$:

$$E_t [r^K_{t+1} + (1 - \delta)] = 1 + r_t,$$

where $r^K_t$ denotes the marginal product of capital, i.e.,

$$r^K_t K_{t-1} = \alpha Y_t.$$ (20)

The FOC with respect to labor input in period-$t$ production is given by

$$w_t N_t = (1 - \alpha) Y_t.$$ (21)

### 2.4 Equilibrium in the Federal Funds Market

Recall that the banking sector has unit mass. Hence, expected deposits of each individual bank, $E_t d_t(i)$, equal aggregate deposits $D_t$ and, similarly, $z_t(i) = Z_t \forall i$. The assumption of homogeneous banks yields the following proposition.

**Proposition 1** If banks are ex ante identical and share a common information set when making their decisions, then each bank $i$ expects the same $E_t d_t(i) = D_t \equiv \int_0^1 d_t(i) di$ and chooses the same $z_t(i) = Z_t \equiv \int_0^1 z_t(i) di$.

The proof of Proposition 1 corresponds to a straightforward symmetry argument. All banks solve an identical optimization problem under idiosyncratic uncertainty.

Suppose further that, after the stochastic deposit inflows, bank $i$ may access the federal funds market with an exogenous probability $\xi_t$ à la Calvo (1983), regardless of its $d_t(i)$. Ignoring transaction costs, interbank borrowing and lending is strictly preferable to “financial autarky” at any interest rate $r^B_t \in (r^K_t, r^L_t)$, i.e., interest payments on reserve balances set a lower bound to the federal funds rate (compare Ennis and Keister, 2008; Keister and McAndrews, 2009; Bech and Klee, 2011).}

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9Limited market participation can arise from private information about idiosyncratic liquidity shocks, as in Hellwig (1994) and von Thadden (1997), or from an agency problem along the lines of Gertler and Kiyotaki (2011), which allows banks to divert interbank credit and prevents perfect liquidity insurance. Although asymmetric access to the interbank market, i.e. $\xi_t$ conditional on $d_t(i)$, seems more realistic, symmetry is a common assumption (see Gertler and Kiyotaki, 2011).

10In reality, banks in need of liquidity can borrow from the Fed through the discount window. The interest rate interval is therefore bounded above by $\min \{r^K_t, r^H_t\}$, where $r^H_t$ is the cost associated with borrowing at
If $\xi_t = 0$, each bank $i$ is stuck with its bank capital $Z_t$ and its deposit realization $d_t(i)$. Its lending to entrepreneurs is thus bounded above by the minimum of its leveraged capital and loanable funds, i.e. $l_t(i) = \min\{\bar{\kappa}Z_t, d_t(i) + Z_t\}$, while its excess liquidity amounts to $rr_t(i) = \max\{d_t(i) - (\bar{\kappa} - 1)Z_t, 0\}$.

If $\xi_t = 1$, the federal funds market allows banks with a high deposit realization to lend excess liquidity and banks with a low realization to narrow their liquidity gap, effectively reallocating funds to their most productive use (compare Keister and McAndrews, 2009). While actual lending to entrepreneurs derives from the Leontief technology in (1), bank-specific excess liquidity amounts to a smaller residual than in isolation:

$$rr_t(i) = \max\{d_t(i) - (\bar{\kappa} - 1)Z_t, 0\}.$$ (22)

Now consider bank $i$ immediately after its idiosyncratic deposit realization:

**Case 1:** If $d_t(i) \leq (\bar{\kappa} - 1)Z_t$, the bank is a potential borrower in the federal funds market. For $r_t^B \in (r_t^R, r_t^L)$, it will demand exactly the amount of federal funds necessary to close its liquidity gap, $(\bar{\kappa} - 1)Z_t - d_t(i)$. As a consequence, the rationing argument in lending to entrepreneurs is bank $i$’s disposable liquidity, $l_t(i) = d_t(i) + b_t(i) + Z_t$, with

$$b_t(i) = \begin{cases} (\bar{\kappa} - 1)Z_t - d_t(i) \geq 0 & \text{with probability } \xi_t \\ 0 & \text{with probability } 1 - \xi_t \end{cases}$$ (23)

**Case 2:** If $d_t(i) > (\bar{\kappa} - 1)Z_t$, bank $i$ is a potential lender in the federal funds market and will offer exactly its excess liquidity, $d_t(i) - (\bar{\kappa} - 1)Z_t$, as long as the federal funds rate is within the boundaries set by the interest paid on reserve balances and the rate earned on loans. In this case, the minimum capital requirement represents the rationing argument in the bank’s Leontief loan production function, i.e. $l_t(i) = \bar{\kappa}Z_t$, and

$$b_t(i) = \begin{cases} (\bar{\kappa} - 1)Z_t - d_t(i) < 0 & \text{with probability } \xi_t \\ 0 & \text{with probability } 1 - \xi_t \end{cases}$$ (24)

Note that, at a federal funds rate $r_t^B < r_t^R$, it is profitable – even for a potential lender – to borrow in the federal funds market and hold the proceeds as reserves in order to make a riskless profit. The increased demand for federal funds should raise their price, i.e. the federal funds rate, until $r_t^B \geq r_t^R$ is satisfied again.

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The discount window, including possible stigma effects. Bech and Klee (2011) develop a model of a segmented interbank market to explain the phenomenon that the effective federal funds rate has been below the interest rate paid on reserve balances in the U.S. since October 2008.
Similarly, \( r_t^B > r_t^L \) implies that lending in the federal funds market is more profitable than making loans to entrepreneurs. Even under the assumption that the banking sector as a whole is a price taker in the market for bank loans and that \( L_t \equiv \int_0^1 l_t(i)di \) has no effect on \( r_t^f \), the increased supply of federal funds should depress the corresponding interest rate until \( r_t^B \leq r_t^L \).

### 2.4.1 Federal Funds Market Clearing

Building on Wen (2011), who investigates input and output inventory dynamics using a Pareto distribution, I assume a suitable functional form for the probability distribution of the stochastic deposit realizations, \( d_t(i) \). This facilitates aggregating the kinked equilibrium conditions of individual banks for the banking sector as a whole.

Denote by \( f(d_t(i)) \) and \( F(d_t(i)) \) the probability density function (pdf) and the cumulative distribution function (cdf), respectively, of bank \( i \)'s stochastic deposit realization and suppose that \( d_t(i) \) is drawn from a time-varying normal distribution with mean \( D_t \) and standard deviation \( \sigma_t \), i.e. \( d_t(i) \sim N(D_t, \sigma_t^2) \). For a continuum of financial firms, \( F(x) \) denotes both the ex ante probability that the deposit realization of a given bank is below \( x \) and the ex post fraction of banks with \( d_t(i) \leq x \).

Federal funds market clearing requires that the supply of liquidity by potential lenders with access to the interbank market equals the demand by potential borrowers. From (23) and (24), it is straightforward to derive aggregate interbank borrowing and lending as the conditional expectation of \( b_t(i) \) for the continuum of banks left and right of the cutoff, \( (\bar{k} - 1)Z_t \), respectively.

Under the distributional assumptions for \( d_t(i) \), aggregate demand for federal funds equals

\[
E_t \left[ b_t(i) \mid d_t(i) \leq (\bar{k} - 1)Z_t \right] = \xi_t \left\{ [(\bar{k} - 1)Z_t - D_t] F((\bar{k} - 1)Z_t) + \sigma_t^2 f((\bar{k} - 1)Z_t) \right\} \geq 0, \tag{25}
\]

while aggregate supply of federal funds by banks with a high deposit realization equals

\[
E_t \left[ b_t(i) \mid d_t(i) > (\bar{k} - 1)Z_t \right] = \xi_t \left\{ [(\bar{k} - 1)Z_t - D_t] [1 - F((\bar{k} - 1)Z_t)] - \sigma_t^2 f((\bar{k} - 1)Z_t) \right\} \leq 0. \tag{26}
\]

The expressions in (25) and (26) yield the federal funds market clearing condition,

\[
\xi_t \left\{ [(\bar{k} - 1)Z_t - D_t] F((\bar{k} - 1)Z_t) + \sigma_t^2 f((\bar{k} - 1)Z_t) \right\} = \xi_t \left\{ [D_t - (\bar{k} - 1)Z_t] [1 - F((\bar{k} - 1)Z_t)] + \sigma_t^2 f((\bar{k} - 1)Z_t) \right\}, \tag{27}
\]

which implies the following important proposition.

**Proposition 2** Assuming a symmetric probability distribution for the stochastic deposit realizations and symmetric access to the federal funds market, \( (\bar{k} - 1)Z_t = D_t \) is necessary and sufficient for an equilibrium in the federal funds market in period \( t \).
Proof. See Appendix A. ■

The federal funds market clears, if and only if banks choose the effective level of bank capital, \( \bar{\kappa}Z_t \), in accordance with expected disposable funds \( (D_t + Z_t) \), before idiosyncratic uncertainty is resolved.

Note that the equilibrium condition in (27) is independent of the federal funds rate, \( r_t^B \). The symmetry assumptions in Proposition 2 imply that the federal funds rate is indeterminate, while \( r_t^B \in (r_t^R, r_t^L) \). In order to pin down \( r_t^B \) uniquely, further assumptions would be necessary.\(^{11}\)

From (2), \( r_t^B \) matters for the distribution of profits across individual banks, whereas the total volume of interbank borrowing and lending is fully determined by the \( f(d_t(i)) \) and \( \xi_t \). Since the ex-post nonzero profits or losses of individual banks, \( \pi_t^b(i) \), are settled through the banker household, the exact federal funds rate is irrelevant for the interest rate spread in financial intermediation. Each bank \( i \) starts the subsequent period with zero net worth.\(^{12}\)

This simplifying assumption also implies that, in normal times, monetary policy merely guarantees a federal funds rate satisfying \( r_t^B \in (r_t^R, r_t^L) \). A monetary extension of the model readily accommodates a Taylor-type interest rate rule, where the central bank sets, e.g., \( r_t^R \) or the interest rate on bank deposits, \( r_t^D \in (r_t^R, r_t^L) \).

2.4.2 Aggregate Lending and Excess Reserves

Given the distributional assumption about \( d_t(i) \), we can furthermore aggregate the actual lending to entrepreneurs by individual banks. Recall that, for banks with a low deposit realization relative to the predetermined amount of bank capital, \( l_t(i) = d_t(i) + b_t(i) + Z_t \), whereas banks with a high deposit realization lend \( l_t(i) = \bar{\kappa}Z_t \).

**Corollary 1** Ex ante expected lending to entrepreneurs by each individual bank \( i \) and ex post lending to entrepreneurs by the banking sector as a whole, \( L_t \), are both given by

\[
E_t l_t(i) = L_t = \bar{\kappa}Z_t - (1 - \xi_t) \left\{ [((\bar{\kappa} - 1)Z_t - D_t) F((\bar{\kappa} - 1)Z_t) + \sigma^2 f((\bar{\kappa} - 1)Z_t)] \right\}.
\]  

(28)

**Proof.** See Appendix A. ■

Note from (28) that, in general, the total volume of loans, \( L_t \), differs from the maximum volume of loans facilitated by effective bank capital, \( \bar{\kappa}Z_t \), implying Corollary 2.

**Corollary 2** With idiosyncratic uncertainty about banks’ stochastic deposit realizations, aggregate lending to entrepreneurs by the banking sector as a whole is bounded above by both arguments

\(^{11}\)Bech and Klee (2011) model bilateral federal funds rates as bargaining outcomes between potential lenders and potential borrowers, reflecting their alternative options and respective bargaining power.

\(^{12}\)In Gertler and Kiyotaki (2011), banks carry non-zero net worth across periods. In order to eliminate differences in net worth and ex ante expected rates of return between investing and non-investing islands, they allow for arbitrage at the beginning of each period, temporarily breaking market segmentation.
of the Leontief loan production technology in (1), i.e.
\[ L_t \leq \min \{ \bar{\kappa}Z_t, D_t + Z_t \}, \] (29)

which holds with equality if and only if \( \xi_t = 1 \).

**Proof.** The proof follows directly from Proposition 2 (\( \bar{\kappa}Z_t = D_t + Z_t \)) and Corollary 1:
\[ L_t = \bar{\kappa}Z_t - (1 - \xi_t)\sigma_t^2 f((\bar{\kappa} - 1)Z_t) = \bar{\kappa}Z_t - (1 - \xi_t)\sigma_t^2 f(D_t). \]

If \( \xi_t < 1 \), idiosyncratic deposit shocks cause an inefficient distribution of liquidity across banks. Note that the financial frictions in this model are literally due to limited market participation.\(^{13}\)

The federal funds market pools liquidity and helps to prevent a wasteful accumulation of deposits in unconstrained financial intermediaries. Only if all banks have access to the federal funds market, liquid reserves are fully drawn down and the maximum leverage ratio is exhausted, while \( \kappa_t \equiv \frac{L_t}{Z_t} \leq \bar{\kappa} \) in general. In line with Holmström and Tirole (1998), \( \xi_t = 1 \) attains an optimal reallocation of loanable funds.

Equation (22) implies that banks with a high deposit realization and no access to the federal funds market end up holding liquidity. This corresponds to the excess reserves of bank \( i \), in the model. Although this treatment of excess reserves might seem somewhat unconventional, it reflects the precautionary motive of banks. In the presence of positive opportunity costs, excess reserves will be zero, unless liquidity shocks and frictions in the interbank market coincide, i.e. \( \sigma_t > 0 \) and \( \xi_t < 1 \) (see, e.g., Ennis and Keister, 2008; Ashcraft et al., 2011; Ennis and Wolman, 2012). As pointed out by Kaufman and Lombra (1980), the observed level of excess reserves is the result of stochastic deposit in- and outflows rather than the result of a desire of banks to hold excess reserves. Finally, i.i.d. liquidity shocks and the Calvo probability of access to the interbank market imply that the realizations of \( rri_t(i) \) are also i.i.d. across banks and through time. This is consistent with the empirical finding of substantial fluctuations in bank-specific reserve balances from quarter to quarter (see Ennis and Wolman, 2012).

Aggregate excess reserves in the banking sector correspond to the conditional expectation of (22) with a probability weight of \( (1 - \xi_t) \), i.e.
\[ E_t rri_t(i) = R_t = (1 - \xi_t) \left\{ \left[ (\bar{\kappa} - 1)Z_t - D_t \right] \left[ 1 - F((\bar{\kappa} - 1)Z_t) \right] + \sigma_t^2 f((\bar{\kappa} - 1)Z_t) \right\} \] (30)
or, equivalently, \( R_t = D_t + Z_t - L_t \).

**Corollary 3** If the federal funds market is in equilibrium, then aggregate excess reserves in the banking sector are bounded below by 0 and bounded above by \( \sigma_t^2 f((\bar{\kappa} - 1)Z_t) \).

\(^{13}\)With a continuum of banks, \( \xi_t < 1 \) can also be interpreted as a restriction on the size of individual interbank loans in the presence of private information, as in Bhattacharya and Fulghieri (1994).
Proof. The proof follows immediately from equation (30) and Proposition 2:

\[ R_t = (1 - \xi_t)\sigma_t^2 f((\bar{\kappa} - 1)Z_t) = (1 - \xi_t)\sigma_t^2 f(D_t). \]

In the model, the total quantity of reserves is determined by the distribution of liquidity and the interbank market friction, \( \xi_t \). This implies that the demand curve for reserves is vertical, while changes in demand are accommodated by the Fed, and seems to contradict the perception that the total level of reserves in the banking system is determined almost exclusively by the central bank (compare Keister and McAndrews, 2009). On the one hand, this reflects the reduced-form approach to modeling liquidity shocks and the exogenous probability of access to the federal funds market, and should hence be taken with a grain of salt. On the other hand, the European Central Bank (ECB) provided unlimited credit at the policy rate through its fixed-rate tender with full allotment (FRFA), starting in October 2008, thus effectively abandoning the control of aggregate reserves outstanding. In section 5, I show how large reserve balances can arise as a byproduct of unconventional monetary policy in the model, whereas \( R_t \) is (close to) zero in normal times (see, e.g., Keister and McAndrews, 2009; Ennis and Wolman, 2012).

The banking sector is closed by assuming free entry into financial intermediation. Thus, expected profits of individual banks and realized profits of the banking sector as a whole equal zero.

2.4.3 The Role of \( \sigma_t \) vs. \( \xi_t \)

At this point, it seems advisable to highlight the difference between the two time-varying parameters that determine the model’s financial friction. The standard deviation of individual banks’ stochastic deposit realizations, \( \sigma_t \), represents the degree of idiosyncratic uncertainty in financial intermediation. While \( r_t^B \in (r_t^R, r_t^L) \), “liquidity insurance” through the federal funds market is strictly preferable to retaining excess reserves or bank capital. Accordingly, \( \sigma_t \) determines the average bank’s demand for interbank borrowing and lending and, for a banking sector of mass one, the aggregate demand for federal funds transactions.

For a given level of idiosyncratic uncertainty, \( \xi_t \) is the ex ante probability that bank \( i \) enters the federal funds market, conditional on \( r_t^B \in (r_t^R, r_t^L) \). With a unit mass continuum of financial intermediaries, \( \xi_t \) also equals the ex post fraction of banks with access to interbank borrowing and lending. It determines thus the fraction of excess liquidity that banks with a high deposit realization lend to banks with a low realization and is best understood as a measure of supply in the federal funds market.

In short, \( \sigma_t \) determines the aggregate volume of excess liquidity, while \( \xi_t \) determines how this is split between federal funds and excess reserves. Using the terminology of Afonso and Lagos (2012), \( \sigma_t \) and \( \xi_t \) reflect the intensive margin and the extensive margin of interbank borrowing
and lending.

Figure C.1 in Appendix C illustrates the implications of $\xi_t$ for aggregate lending to entrepreneurs in terms of $f(d_t(i))$. Consider panel (a) first. If $\xi_t = 1$, interbank borrowing and lending provides perfect insurance against idiosyncratic deposit shocks. Each bank $i$ attains the optimal level of liquidity, $d_t(i) + b_t(i) = (\bar{\kappa} - 1)Z_t$, regardless of its realization $d_t(i)$. Corollary 1 implies that $L_t = D_t + Z_t = \bar{\kappa}Z_t$, in this case, while $R_t = D_t + Z_t - L_t = 0$.

Consider now panel (b). If $\xi_t < 1$, only a fraction of all banks has access to the interbank market. Accordingly, $d_t(i) + b_t(i) \neq (\bar{\kappa} - 1)Z_t$ for some banks $i$, due to $b_t(i) = 0$. For a unit-mass continuum of banks, $\xi_t$ can equivalently be interpreted as the share of each bank’s $|d_t(i) - (\bar{\kappa} - 1)Z_t|$ that is offset by borrowing or lending in the federal funds market (compare Bhattacharya and Fulghieri, 1994). In aggregate, this corresponds to the shaded area in panel (b), while aggregate excess reserves correspond to the non-shaded area below $f(d_t(i))$ right of the cutoff $(\bar{\kappa} - 1)Z_t$. In this case, $L_t < D_t + Z_t = \bar{\kappa}Z_t$ and $R_t > 0$.

3 Calibration and Steady State

The model presented above is highly stylized in many dimensions. As it is therefore impossible to match the empirical counterparts of all variables, the following calibration focuses on the banking sector and, in particular, on aggregate excess reserves and the federal funds market.

3.1 Parameter Values

As far as possible, I choose standard parameter values in line with the RBC literature. The worker household’s inverse Frisch elasticity of labor supply, $\gamma$, is set equal to 1. The weight of labor in the utility function, $a$, is calibrated in order to obtain a steady-state employment equal to $1/3$ of the total time endowment.

Note that bankers are assumed to be less patient than workers. Their subjective discount factor, $\beta_b$, is set to .94, whereas $\beta_w = .995$ for the worker household, implying a quarterly rate of time preference of 6.4% and .5%, respectively. By setting the quarterly depreciation rate of physical capital, $\delta$, to .025 and the elasticity of final output with respect to capital, $\alpha$, to $1/3$, the capital- and goods-producing sectors in my model are entirely standard.

I now turn to the calibration of the banking sector. In equation (1), $\bar{\kappa}$ represents the maximum ratio of bank loans to bank capital. With regard to the Basel accords, I set $\bar{\kappa}$ to 12.5, implying a minimum ratio of bank capital to entrepreneurial loans of 8%. In the presence of excess reserves, $\frac{1}{\bar{\kappa}t} = \frac{Z_t}{L_t}$ will be higher than this regulatory minimum. Note also that this implies an aggregate debt-to-equity ratio, $\frac{D_t}{Z_t}$, of $(\bar{\kappa} - 1) = 11.5$, in equilibrium.
Recall that $\sigma_t$ and $\xi_t$ denote the exogenous standard deviation of stochastic deposit realizations and the Calvo probability of access to the federal funds market. In the benchmark calibration, the corresponding stationary values, $\sigma^*$ and $\xi^*$, are set to 1, targeting the situation before the crisis, when interbank lending was positive and excess reserves were virtually equal to zero.

The dynamics of the general equilibrium model are driven by four stochastic processes. I assume that innovations to goods production technology, consumer preferences, the variance of deposit realizations, and participation in the federal funds market follow a first-order autoregressive process in logs, i.e., $\ln(x_t) = \rho_x \ln(x_{t-1}) + \varepsilon^x_t$, where $\rho_x = .95$ for $x = A, \zeta, \sigma, \xi$. For convenience, Table 1 summarizes the parameters determining the steady state.

### 3.2 Stationary Equilibrium

Abstracting from aggregate shocks, it is straightforward to solve for the stationary equilibrium. Note that banks are subject to idiosyncratic uncertainty about their deposit realizations, even in the absence of aggregate risk.

The above calibration implies an annualized capital-output-ratio of 2.4. Consumption of the worker and banker households account for 71.1 and 4.9%, respectively, while capital investment accounts for the remaining 24% of total output.

Due to the fact that entrepreneurs require bank loans for purchasing the physical capital stock, the aggregate loan-to-output ratio is identical to the capital-output ratio. Note that $L^* = D^* + Z^*$, if and only if all banks have access to the federal funds market, i.e. $\xi^* = 1$, as in the benchmark case. Hence, banks can perfectly insure against liquidity shocks by borrowing and lending amongst each other.

A steady-state value of interbank borrowing, $B^*|_{d_t(i) \leq (\bar{\kappa} - 1)Z^*}$, equal to 4.4% of aggregate deposits and the absence of excess reserves in the steady state ($R^* = 0$) were targeted by setting $\sigma^* = \xi^* = 1$ in the benchmark calibration. These values are understood to be only suggestive. By setting the maximum leverage ratio, $\bar{\kappa}$, to 12.5, I ensure that the equity-to-loans ratio, $\frac{Z^*}{L^*}$, matches the minimum capital requirement of 8% stipulated in the Basel accords.

The quarterly interest rate on bank deposits, $r^{D^*}$, is pinned down by the subjective time discount rate of workers and invariant to the rest of the calibration. Similarly, bankers’ subjective time discount rate determines the steady-state return on bank capital, $r^{Z^*} = .0638$. Free entry into financial intermediation implies that banks make zero profit, on average, whereas each bank
i makes zero profit, if and only if $\xi^* = 1$. In general, microeconomic uncertainty implies that individual banks realize losses or gains, which are netted through the banker household at the end of each period. Given the steady-state values of the other variables in equation (2), the free-entry condition also determines the loan interest rate, $r^L = .0097$.

The user cost of capital comprises the loan rate and the quarterly depreciation rate. Assuming perfect competition, this will be equal to the marginal product of capital, i.e. $r^K = .0347$ in the benchmark steady state.

A ceteris paribus increase in the standard deviation of deposit realizations, $\sigma^*$, implies a higher steady-state volume of federal funds, without affecting $R^* = 0$. A ceteris paribus decrease in $\xi^*$ reduces banks’ participation in the interbank market and raises their holdings of excess reserves, instead. As a consequence, the equity-to-loans ratio rises, signalling higher costs of financial intermediation, which are passed on to entrepreneurs through an increase in the interest rate on bank loans, $r^L$. Accordingly, a decrease in $\xi^*$ impairs the efficiency of the banking sector. Note that the debt-to-equity ratio, $D^*Z^* = 11.5$, will not be affected. The set of steady-state values obtained for the benchmark parameter calibration is summarized in Table 2.

4 Shocks to Financial Intermediation

This section discusses the implications of idiosyncratic uncertainty in financial intermediation for the propagation of two novel shocks inherent in the banking sector. For the dynamic analysis, the model is loglinearized around the non-stochastic steady state (see Appendix B) and impulse response functions are computed for $\xi^* = 1, .5, and 0$. While most variables’ steady state values are broadly insensitive to $\xi^*$, $B^*$ and $R^*$ are equal to $(\sigma^*)^2f((\bar{\kappa} - 1)Z^*)$ and 0 for $\xi^* = 1$ and vice versa for $\xi^* = 0$, respectively. For $\xi^* = .5$, half of all banks borrow or lend in the interbank market. The liquidity risk $(\sigma^*)^2f((\bar{\kappa} - 1)Z^*)$ is therefore split equally between federal funds and excess reserves. For ease of interpretation, the impulse response functions of federal funds and excess reserves are in terms of deviations from steady state as a fraction of steady-state deposits, $D^*$.

The propagation of standard supply and demand shocks will not be discussed here, in detail. Nevertheless, two comments are in order. First, the impulse responses of real variables to an innovation in total factor productivity (TFP) and household preferences, respectively, are in line with those obtained from a textbook RBC model. Second, $\xi^*$ is irrelevant for the propagation of standard supply and demand shocks. In response to a positive TFP shock, e.g., the banking sector’s aggregate balance sheet expands to accommodate higher loan demand by entrepreneurs.

14In particular, maintaining $\xi^* = 1$ and thus unrestricted access to the federal funds market.
The assumption that \( \sigma_t \) follows an exogenous process implies that the uncertainty faced by each bank \( i \) is independent of the size of its balance sheet. Accordingly, an expansion corresponds to a rightward shift of \( f(d_t(i)) \), or a \textit{variance-preserving increase in the mean}. Similarly, a positive preference shock induces a contraction of banks’ balance sheets and a leftward shift of \( f(d_t(i)) \). The corresponding impulse response functions are available from the author.

### 4.1 Uncertainty Shocks

Borrowing from the terminology in Bloom (2009), I use the term “uncertainty shock” to describe an exogenous change in the standard deviation of banks’ idiosyncratic deposit realizations. This is adequate, as changes in \( \sigma_t \) represent second-order shocks, i.e. innovations in the variance of \( d_t(i) \) for a constant mean. Assuming a symmetric probability distribution, liquidity risk in this model is inherently microeconomic, and should not be confounded with the growing literature on macroeconomic volatility shocks (see, e.g., Fernández-Villaverde et al., 2011).\(^{15}\)

Figure 2 plots the impulse response functions to an unexpected transitory 10% increase in the standard deviation of \( d_t(i) \), i.e., \( \sigma_t \) jumps from 1 to 1.1. While this corresponds to a moderate increase in the variance of \( d_t(i) \) by 21%, the reported impulse responses are scalable by changing the size of the shock, given the model’s loglinear approximation around the steady state.

Recall that the capital stock is acquired and installed one period in advance of production. Therefore, shocks to financial intermediation affect the firm’s capacity with a lag of one quarter. A temporary increase in \( \sigma_t \) implies that, on average, banks experience larger deviations from \( E_t d_t(i) \) and have thus a stronger need for federal funds transactions. Otherwise, they end up holding larger excess reserves.

Intuitively, this kind of uncertainty shock should not affect real economic activity, if access to the federal funds market is complete. Figure 2 illustrates that, by borrowing from and lending to each other, banks can perfectly insure against idiosyncratic uncertainty, regardless of \( \sigma_t \), as long as \( \xi^* = 1 \). The volume of interbank borrowing and lending increases by 0.43% of steady-state deposits, before converging back to its steady state, whereas none of the other variables deviates from its stationary equilibrium.\(^{16}\) Thus, a frictionless federal funds market completely absorbs uncertainty shocks.

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\(^{15}\)The deviations of all banks’ stochastic deposit realizations from the mean of the distribution average out, in each period. I assume that there is no uncertainty about the aggregate level of deposits, \( D_t \).

\(^{16}\)Note that a percentage deviation i. t. o. steady-state deposits, i.e. \( \frac{B_t - B^*}{B^*} \), of 0.43% corresponds to a percentage deviation from steady-state interbank borrowing, \( \frac{B_t - B^*}{B^*} = \frac{B_t - B^*}{B^*} \cdot \frac{B^*}{D^*} = 0.43 \cdot \frac{1}{0.043} \), of 10%.
Consider now $\xi^* = .5$, where bank’s liquidity risk is split equally between federal funds and excess reserves, in the steady state. Even though both bank capital and deposits increase in response to the $\sigma_t$-shock, aggregate lending to entrepreneurs decreases. The wedge between $D_t + Z_t$ and $L_t$ corresponds to the increase in excess reserves. Both $B_t$ and $R_t$ increase, if access to the federal funds market is limited. While interbank borrowing and lending is costless, however, higher reserves induce resource costs, which are passed on to entrepreneurs through the interest rate on bank loans. The spread in financial intermediation increases, as the latter rises by more than the interest rate on deposits.

From equation (19), an increase in the loan rate, $r^L_t$, must be offset by an equivalent increase in the expected marginal product of the physical capital stock. With TFP unchanged, this implies a reduction in $K_t$, which becomes productive as of period $t + 1$. Accordingly, capital investment drops, on impact, before recovering in the second quarter after the shock. Initially, the worker household raises its labor supply, as the higher interest rate on bank deposits induces higher savings, less consumption, and thus an increase in the marginal utility of consumption. The additional labor input temporarily sustains the original output level.\(^\text{17}\)

Once the downsized capital stock becomes productive, the economy enters into a recession. The decrease in $K_t$ reduces the marginal productivity of labor, both $N_t$ and $w_t$ drop below their steady-state values, and output falls short of its stationary equilibrium value by about 8 basis points, before slowly recovering along with the capital stock and employment.

If $\xi^* = 0$, the federal funds market is inoperative. As a consequence, banks with a high deposit realization, $d_t(i) > (\bar{\kappa} - 1)z_t(i)$, maintain excess reserves, while banks with a low realization, $d_t(i) < (\bar{\kappa} - 1)z_t(i)$, lack the liquidity to exhaust the minimum capital requirement and end up with $\frac{z_t(i)}{d_t(i)} > 8\%$. The same frictions discussed in detail for $\xi^* = .5$ occur to an extreme degree, in this case. To be precise, the impulse responses to an uncertainty shock in terms of the deviations from steady state are twice as large for $\xi^* = 0$.

In terms of the model, the period before Lehman Brothers can be interpreted as a period of pure uncertainty shocks, while $0 < \xi_t < 1$. Ashcraft et al. (2011) find that both aggregate excess reserves and aggregate federal funds transactions increased in August 2007, consistent with the conditional comovement of $B_t$ and $R_t$ in the model.\(^\text{18}\)

A frictionless interbank market allows banks to perfectly insure against idiosyncratic uncertainty

\(^{17}\)Recall that the economy’s aggregate resource constraint deviates from the standard $Y_t = C_t + I_t$. Instead, in this model, $Y_t = C^*_t + C^{Z_t}_t + I_t + (Z_t - Z_{t-1}) + (R_t - R_{t-1})$, i.e., changes in the stock of bank capital, $Z_t$, or in aggregate excess reserves, $R_t$, absorb or release real resources, corresponding to a saving or storage technology. In the steady state, the resource constraint boils down to $Y^* = C^* + I^*$, where $C^* = C^{m*} + C^{b*}$.

\(^{18}\)Ashcraft et al. (2011) argue that, in response to the increase in liquidity risk, “constrained banks”, which are not able to borrow in the federal funds market, could increase their holdings of precautionary excess reserves, whereas banks with the ability to borrow could tap the federal funds market as needed, i.e. $\sigma_t \uparrow$ for $\xi_t = \xi^* < 1$.\(^\text{20}\)
as well as $\sigma_t$-shocks. The lower $\xi^*$, the larger is the share of all banks with either excess reserves or excess bank capital. This impairs the efficiency of financial intermediation in the steady state and, more importantly, the banking sector’s ability to absorb the effects of uncertainty shocks on real economic activity.

4.2 Financial Friction Shocks

Consider now a temporary exogenous change in the fraction of financial institutions participating in the federal funds market. Events of this kind are called “illiquidity waves” in Brunnermeier (2009) or “liquidity crises” in Gertler and Kiyotaki (2011). Recall that idiosyncratic uncertainty ($\sigma_t > 0$) induces a demand for liquidity insurance, whereas $0 \leq \xi_t \leq 1$ determines the relative prevalence of federal funds and excess reserves.

$\xi_t$ captures all factors which might keep an individual bank away from the interbank market. The model does not distinguish between voluntary autarky of potential lenders, e.g. due to risk considerations or alternative lending opportunities, and involuntary exclusion of potential borrowers from the market, e.g. due to selection. One interpretation for the exogenous change in $\xi_t$ is the deterioration of banks’ balance sheets following the bankruptcy of Lehman Brothers. The resulting increase in the perceived or actual counterparty risk of interbank credit motivates the voluntary absence of potential lenders and thus the above symmetry assumption.

Figure 3 depicts the impulse response functions of selected variables to a financial friction shock. $\xi_t$ drops by .1 before following an autoregressive process back to $\xi^*$, i.e., ten percent of the unit mass of banks do not participate in the federal funds market, temporarily. As a consequence, a larger fraction of banks ends up holding either excess reserves or excess capital, after realizing an imbalance between $d_t(i)$ and $(\bar{\kappa} - 1)Z_t$.

In response to the shock, aggregate interbank borrowing drops by .43% of steady-state deposits, whereas aggregate excess reserves increase by the same amount. Comparing impulse responses with those in Figure 2, financial friction and uncertainty shocks have similar qualitative effects, except for $B_t$ and $R_t$, which deviate from their steady-state values in opposite directions.

The increase in excess reserves raises the spread between the interest rates on bank loans and deposits. Higher costs of financial intermediation require an offsetting increase in the expected marginal product of capital, i.e. a reduction in $K_t$, and thus a pronounced disinvestment in the

\[\text{Include Figure 3 about here}\]

\[\text{\footnotesize Note that } \hat{\xi}_t \text{ is in percentage point rather than in } \% \text{ deviations from steady state in order to avoid an effect of } \xi^* \text{ on the size of the shock. Admissible values for } \xi_t \text{ are from the interval } [0, 1]. \text{ Accordingly, impulse responses for } \xi^* = 0 \text{ are skipped in Figure 3, as a further deterioration of access to the federal funds market is impossible, in this case.}\]
period of the shock. While the initial capital stock is still in use, production can only adjust through employment. As in section 4.1, the higher interest rate on deposits induces the worker household to save more, consume less, and expand its labor supply, which depresses the real wage and sustains output, initially.

Once the downsized capital stock becomes productive, the economy enters into a recession. Output and employment follow the same qualitative and quantitative patterns as in response to a 10%-increase in $\sigma_t$ for $\xi^* = 0$. Now, however, the impulse responses are independent of $\xi^*$. For a given liquidity risk, what matters is the change in the financial friction, i.e. the share of banks that is “driven out of the federal funds market”, rather than its steady-state level.

In this model, uncertainty and financial friction shocks are distinguishable only by the responses of federal funds and excess reserves. Both $\sigma_t$ and $\xi_t$ affect the spread in financial intermediation and thus real economic activity in the same way. The impulse responses to a financial friction shock are independent of $\xi^*$, as long as the steady-state participation in the federal funds market is irrelevant for the size of the innovation.

The empirical findings in Afonso and Lagos (2012) suggest that the period after October 2008 corresponds to a period of coinciding uncertainty and financial friction shocks. Although average trade size increased, which suggests a positive $\sigma_t$-shock, aggregate federal funds market volume decreased and excess reserves rose dramatically, suggesting a negative $\xi_t$-shock. While the model predicts that both shocks raise $R_t$, if $\xi_t < 1$, the overall impact on $B_t$ is ambiguous.

5 Unconventional Monetary Policy

In the previous analysis, monetary policy is assumed to merely guarantee that the interest rate on federal funds satisfies $r^B_t \in (r^R_t, r^L_t)$. Central bank policy in “normal” times is commonly described as implementing the desired monetary stance by targeting the federal funds rate. As illustrated in Figure 1, the benchmark calibration ($\xi^* = 1$) is consistent with the situation until mid-2007, when excess reserves were virtually equal to zero. A monetary extension of the model would readily accommodate a Taylor-type interest rate rule, where the central bank sets, e.g., $r^R_t$ or the interest rate on bank deposits, $r^D_t \in (r^R_t, r^L_t)$.

Since the beginning of the financial crisis in 2007, however, the U.S. Board of Governors of the Federal Reserve System has introduced various non-standard measures to secure the liquidity of financial intermediaries and to prevent a collapse of the banking sector. These measures can be categorized as direct lending to non-financial firms, equity injections into banks, and liquidity

\[^{20}\text{Recall that the continuum of banks is owned by a representative banker household and that their individual profits and losses net out each period. As a consequence, the distribution of profits between borrowers and lenders in the federal funds market is irrelevant for the general equilibrium.}\]
facilities (see, e.g., Gertler and Kiyotaki, 2011). Due to the focus of the current paper on excess reserves and the federal funds market, this section analyzes the theoretical implications of the Fed’s liquidity facilities – in particular its interest payments on reserve balances and its lending programs.

5.1 Interest on Reserve Balances

On October 6, 2008, the Fed announced that it would start paying interest on required and excess reserve balances in order to “[...] give the Federal Reserve greater scope to use its lending programs to address conditions in credit markets while also maintaining the federal funds rate close to the target established by the Federal Open Market Committee”

In the present model, interest payments on reserves correspond to a temporary or permanent exogenous increase in $r^R_t$. Neglecting general equilibrium effects, it is straightforward to partially differentiate equation (2) with respect to $r^R_t$:

$$\frac{\partial E_t \pi^b_{t+1}(i)}{\partial r^R_t} = E_t r r_t(i) \geq 0.$$

Ceteris paribus, interest payments on excess reserves by the central bank are equivalent to a banking-sector subsidy.

The assumption of free entry into financial intermediation implies that $E_t \pi^b_{t+1}(i) \equiv 0$. Again neglecting general equilibrium effects, we can solve (2) for $r^L_t$ and take the partial derivative with respect to $r^R_t$:

$$\frac{\partial r^L_t}{\partial r^R_t} = -\frac{E_t r r_t(i)}{E_t l_t(i)} \leq 0.$$

Ceteris paribus, $r^L_t$ is a weakly decreasing function of $r^R_t$, if we assume free entry into banking.

The subsidy is passed on to the real economy, as banks demand a lower interest rate on their loans to entrepreneurs. As a consequence, in the presence of financial frictions, interest payments on reserve balances lower the cost of financial intermediation to the goods-producing sector.

5.2 Central Bank Lending

The previous analysis shows that the coincidence of bank liquidity risk and limited federal funds market participation entails financial frictions. Both an increase in $\sigma_t$ (for $\xi_t < 1$) and a decrease in $\xi_t$ (for $\sigma_t > 1$) imply higher excess reserves and thus higher costs of financial intermediation, which are reflected in a larger spread between the interest rates on bank deposits and bank loans.

Unprecedented levels of liquidity and counterparty risk coincided in the interbank market during

---

the financial crisis of 2007–2009 and, in particular, after the bankruptcy of Lehman Brothers (see, e.g., Brunnermeier, 2009; Wu, 2011), warranting thus unconventional monetary policy.

< Include Figure 4 about here >

Figure 4 illustrates how the increase in total reserve balances held at Federal Reserve Banks can be attributed to the Fed’s lending and asset purchase programs (compare Ennis and Wolman, 2012, Figure 2). The former comprised direct lending to banks through the discount window, the newly created Term Auction Facility (TAF)\(^{22}\), and central bank liquidity swaps. Between December 2007 and October 2008, the Fed largely sterilized the reserves added through lending facilities by selling securities in the open market. Following the bankruptcy of Lehman Brothers, sterilization was no longer sustainable and reserves were left to rise. As illustrated in the upper panel of Figure 1, virtually all of this increase raised excess reserves. At the peak of the crisis, during the fourth quarter of 2008, reserve balances added through central bank credit amounted to 18.6% of total bank deposits. For this reason, I will focus on the Fed’s lending programs, when analyzing the implications of unconventional monetary policy, in this model.

Similar to Dib (2010), I assume that, in times of distress, the central bank can inject liquidity directly into the banking sector. In reality, discount window and TAF loans were collateralized. Since there is no role for a fiscal authority, I refrain from introducing government bonds, which could serve as collateral. Note that the absence of default or agency problems between financial intermediaries and their creditors implies that this is not a strong assumption.

Instead, bank \(i\) pays a real interest rate \(r_t^X\) on central bank credit \(x_t(i)\) between period \(t - 1\) and \(t\). The corresponding central bank revenue, \(r_t^X X_t\), where \(X_t \equiv \int_0^1 x_t(i) di\), net of possible interest payments on reserve balances, \(r_t^R R_t\), equals an efficiency cost of unconventional monetary policy, which is assumed to be a deadweight loss to the economy.

The rest of the section compares the stabilizing effects of liquidity injections, implemented in two different ways. Injections are assumed to take place after banks have received their Calvo signal but before the federal funds market opens. Adjusting equations (1)–(3), it is straightforward to see that, while \(R_t \approx X_t\), the efficiency cost is fully determined by the exogenous spread between the policy interest rates on central bank credit and reserve balances, \(r_t^X - r_t^R\).

Suppose that the central bank observes the idiosyncratic deposit realization of each bank \(i\) and whether bank \(i\) participates in the federal funds market. Hence, it directs funds to institutions

\(^{22}\)On December 12, 2007, the Federal Reserve, the ECB, and other central banks announced “measures designed to address elevated pressures in short-term funding markets” (see the press release on http://www.federalreserve.gov/newsevents/press/monetary/20071212a.htm). In the U.S., the TAF was created as a stigma-free alternative to the discount window (see also Ashcraft et al., 2011). Under the TAF, the Fed conducted biweekly auctions of term funds to all depository institutions eligible to borrow under the primary credit program. After the last auction in March 2010, the remaining outstanding loans ran off as scheduled.
with \(d_t(i) < (\bar{\kappa} - 1)Z_t\) and \(b_t(i) = 0\), exclusively. In the following, I will refer to this policy as a “targeted liquidity injection”.

Now suppose that the central bank does not know whether bank \(i\) is liquidity-constrained, i.e., it observes neither the bank’s deposit realization nor the bank’s Calvo signal. As a consequence, it injects liquidity into all banks, regardless of their individual \(d_t(i)\) and \(b_t(i)\). Similar to Gertler and Karadi (2011), I assume that the central bank’s liquidity injection responds to fluctuations in the costs of financial intermediation, according to the feedback rule

\[
X_t = \phi \left[ (r_t^L - r_t^D) - (r_t^L - r_t^D) \right] D, \tag{31}
\]

where \(\phi \geq 0\) denotes the feedback parameter, \(r_t^L - r_t^D\) the steady-state spread, and \(X = 0\), in the steady state. For simplicity, this policy will be called a “broad liquidity injection” below.\(^23\)

During the financial crisis, the Fed repeatedly cut the interest rate on primary discount window credit from 6.25% in July 2007 to .5% in December 2008 and started paying interest on reserve balances from October 2008 onwards. The implied spread between the “discount rate” and the interest rate paid on excess reserves equaled 100 bps p.a. during October 15–29, 60 bps p.a. from October 29 to November 5, and 25 bps p.a. between November 2008 and February 2010. Focusing on the final quarter of 2008, I set the policy spread, \(r_t^X - r_t^R\), to 50 bps p.a., while the feedback parameter \(\phi\) is set to a suggestive value of 800.\(^24\) Table 3 summarizes the parameter values associated with the efficiency cost of central bank credit and the feedback rule in (31).

5.2.1 The Response to an Uncertainty Shock

Figures 5 and 6 plot the impulse response functions to an uncertainty shock, which raises the variance of \(d_t(i)\), with unconventional monetary policy.\(^25\) For ease of comparison, the vertical axis is scaled identically in both figures. The size of the disturbance is the same as in Figure 2.

Consider first the case of a targeted liquidity injection, i.e., the central bank responds to the shock by lending to banks with a low deposit realization \((d_t(i) < (\bar{\kappa} - 1)Z_t)\) and no access to the federal funds market. Figure 5 illustrates that a targeted intervention comes close to stabilizing the interest rate spread in financial intermediation, \(r_t^L - r_t^D\), and mitigates thus the effect of the uncertainty shock on the real economy.

\(^{23}\)It is straightforward to show analytically how targeted and broad liquidity injections by the central bank affect the aggregate volumes of federal funds, lending to entrepreneurs, and excess reserves in the banking sector. All derivations are available from the author upon request.

\(^{24}\)Recall that the efficiency cost of unconventional monetary policy is determined by the spread rather than by \(r_t^X\) and \(r_t^R\), separately. Unless we are interested in the effects of \(r_t^X > 0\), in isolation, it is therefore equivalent to set \(r_t^R = 0\), i.e., \(r_t^X - r_t^R = r_t^X > 0\).

\(^{25}\)For computational reasons, impulse responses are plotted for \(\xi^* = .9, .5, \text{ and } .1\).
The extent to which the spread is stabilized depends on the steady-state federal funds market participation, $\xi^*$. Uncertainty shocks raise the variability of $d_t(i)$ around $D_t$ and thus the desired aggregate volume of interbank borrowing and lending. If a large fraction of banks participates in the federal funds market ($\xi^* = .9$), the disturbance primarily leads to a higher aggregate volume of interbank lending. Only a small share of financial institutions is liquidity-constrained and qualifies thus for central bank credit. At the same time, the fraction of banks with excess liquidity that cannot lend in the federal funds market is relatively small.

As $\xi^*$ decreases, an growing share of all institutions qualifies for a targeted liquidity injection. Accordingly, the shock implies a less pronounced increase in $B_t$ but a more pronounced increase in $R_t$, calling for a higher aggregate injection by the central bank.

Due to the efficiency cost of unconventional monetary policy, a smaller liquidity injection comes closer to stabilizing $r^L_t - r^D_t$ and real economic activity. Accordingly, targeted central bank credit represents an imperfect substitute for the federal funds market. Note that, with zero efficiency costs, the central bank could fully stabilize the spread in financial intermediation. Trivially, no policy intervention is required, if $\xi^* = 1$.

Consider now the case of a broad liquidity injection, i.e., the central bank lends to all banks regardless of their idiosyncratic deposit realization and Calvo signal. Figure 6 illustrates that the broad intervention is less successful in stabilizing the spread in financial intermediation and thus real economic activity, due to the fact that central bank lending to all banks involves efficiency costs, whether they are constrained or not.

Note that there are two opposing effects of $\xi^*$. On the one hand, a lower $\xi^*$ implies that a larger fraction of banks is liquidity-constrained and would therefore benefit from a policy intervention. On the other hand, the “efficiency” of broad relative to targeted liquidity injections increases, as a smaller fraction of $X_t$ goes to unconstrained banks, which do not satisfy the criteria for a targeted injection.\footnote{For $\xi^* = 0$, a broad liquidity injection is half as efficient as a targeted liquidity injection. For $\xi^* \to 1$, the relative efficiency goes to 0.}

Regardless of $\xi^*$, a broad intervention implies an approximate one-for-one increase in excess reserves. First, half of the injection hits banks with a high deposit realization ($d_t(i) > (\kappa - 1) Z_t$). For a fraction $(1 - \xi_t)$ of the latter, $X_t$ only raises the amount of excess liquidity, as they cannot lend in the federal funds market. Second, a fraction $\xi_t$ goes to institutions that would otherwise clear their imbalances amongst each other in the federal funds market, crowding out interbank borrowing and lending. Absent efficiency costs, the central bank could fully stabilize the spread
in financial intermediation by injecting sufficient liquidity into all banks. As a consequence, aggregate excess reserves would increase one-for-one with $X_t$, while interbank borrowing and lending would remain equal to its steady-state value.

A targeted injection only provides liquidity to banks that have experienced an unexpectedly low deposit realization and cannot borrow in the federal funds market. It just restores the liquidity distribution left of the cutoff, before the shock, whereas a broad injection does not discriminate. Figure C.3(a) illustrates that the latter shifts $f(d_i(i) + Z_t + X_t)$ to the right, thus raising excess reserves and crowding out federal funds. The bold black line indicates the effect of interbank borrowing on the distribution of liquidity after the unconventional monetary policy intervention.

### 5.2.2 The Response to a Financial Friction Shock

Figures 7 and 8 plot the impulse response functions to a financial friction shock, which reduces the probability that any bank $i$ participates in the federal funds market, with unconventional monetary policy. For ease of comparison, the scale of the vertical axis is the same across figures. The size of the disturbance is the same as in Figure 3, i.e., $\xi_t$ drops by .1, following an AR(1) process afterwards.

Figure 7 illustrates that a targeted liquidity injection comes close to offsetting the effects of a financial friction shock. Due to the fact that the shock is in percentage points rather than in percent of $\xi^*$, the injection is independent of the steady-state participation in the federal funds market. Comparing impulse responses with those in Figure 3, the intervention does not generate excess reserves beyond those due to the exogenous disturbance, as funds are exclusively directed to liquidity-constrained banks. In terms of $f(d_i(i) + Z_t + X_t)$, the central bank literally “refills” the darker shaded area left of the cutoff in Figure C.2(b), which was lost due to the $\xi_t$-shock.

Figure 8 illustrates the case of a broad liquidity injection. Note that, although the size of the shock is the same as in Figure 7, the size of the intervention and its effect on $B_t$ and $R_t$ varies substantially. This is due to the fact that the “efficiency” of broad relative to targeted liquidity injections depends on the federal funds market participation.

The closer $\xi_t$ is to 1, the smaller is the fraction $.5(1 - \xi_t)$ of institutions qualifying for a targeted liquidity injection, and the larger is thus the fraction $1 - .5(1 - \xi_t)$ of $X_t$ going to unconstrained banks. For $\xi^* = .5$, e.g., a broad injection must be four times larger than a targeted injection, as a quarter of all banks is liquidity-constrained. For $\xi^* = .9$, the former must be 20 times the size
of the latter, as only 5% of all banks are liquidity-constraint. As a consequence, broad liquidity injections entail a sizeable accumulation of excess reserves in the banking sector. Moreover, a multiple of interbank borrowing and lending is crowded out, relative to the shock itself.

The assumption of efficiency costs in central bank lending implies that the spread in financial intermediation cannot be fully stabilized. Accordingly, the impact of a financial friction shock on real economic activity is larger than in Figure 7, if monetary policy follows the feedback rule in equation (31). Absent efficiency costs, the central bank could fully stabilize \( r_t^L - r_t^D \) around its steady-state value by injecting liquidity into all banks. As a consequence, aggregate excess reserves would increase one-for-one with \( X_t \), while federal funds transactions would temporarily be crowded out almost completely, given the size of the \( \xi_t \)-shock.

The dark shaded area in Figure C.3(b) indicates the effect of a broad liquidity injection on the probability density function of bank-specific liquidity. \( f (d_t(i) + Z_t + X_t) \) shifts to the right in order to compensate for the medium shaded area left of the cutoff. The bold black line illustrates the scope for interbank borrowing after the central bank’s intervention. Note that the slope of \( f [d_t(i) + b_t(i) + Z_t + X_t|\xi = .4] \) is flatter than the slope of \( f [d_t(i) + b_t(i) + Z_t|\xi = .5] \), because the broad injection moves all banks with \( d_t(i) < (\bar{\kappa} - 1)Z_t \) closer to the cutoff.

Although Gertler and Kiyotaki (2011) focus on direct credit intermediation by the central bank, there are two interesting parallels between their results and mine. First, an increase in central bank lending crowds out private credit on non-investing islands one-for-one in the same way that central bank liquidity injections crowd out federal funds. Second, a given level of intermediation is therefore more effective in relaxing financial constraints, when targeted to investing islands.

5.3 The Model and the Financial Crisis of 2007–2009

The theoretical analysis in sections 4 and 5 suggests that both uncertainty and financial friction shocks can trigger a recession, while liquidity injections by the monetary authority provide an imperfect substitute for interbank credit. The effectiveness of policy interventions in response to \( \sigma_t \)- and \( \xi_t \)-shocks depends on the efficiency cost of central bank credit, \( (r_t^X - r_t^R)X_t \). Accordingly, the central bank can “enhance” the intervention by lowering the discount rate, \( r_t^X \), or by paying interest on excess reserves, \( r_t^R \). This captures a good deal of the measures taken by the Fed after the start of the crisis in August 2007 and, in particular, after the failure of Lehman Brothers in September 2008. The creation of the TAF in order to alleviate the stigma of borrowing at the discount window can be interpreted as an attempt to lower the perceived cost of liquidity assistance even further.

27Recall that the increase in excess reserves is due to (i) the shock itself, (ii) .5 \((1 - \xi_t)\) of the injection going to banks with excess liquidity, beforehand, and (iii) \( \xi_t \) going to banks with access to the federal funds market.
Why are there hardly any excess reserves before the failure of Lehman Brothers? According to the model, in the presence of positive opportunity costs, \( R_t = 0 \) while \( \xi_t = 1 \), and the federal funds market reallocates liquidity between financial intermediaries. The unprecedented increase of excess reserves in October 2008 coincided with tensions in the interbank market, a drop in the federal funds rate, and interest on reserve balances. This simultaneously raised the benefit and reduced the opportunity cost of holding excess reserve (compare Ennis and Wolman, 2012). Figure 4 shows that, while the opportunity cost was positive, the Fed sterilized the increase in reserves through open market sales in order to keep the federal funds rate close to the target rate. As short-term interest rates approached zero in late 2008, the Fed suspended the sterilization of excess reserves added through its lending facilities.

Do large excess reserves imply that unconventional monetary policy has been ineffective? Keister and McAndrews (2009) argue that the quantity of reserves reflects the size of the Fed’s initiatives rather than its effectiveness. Similarly, excess reserves in the model are a by-product and rise almost one-for-one with central bank credit. Without unconventional monetary policy, i.e. at a positive opportunity cost, excess reserves signal financial frictions and banks’ reluctance to lend in the interbank market (see, e.g., Ashcraft et al., 2011). At a zero opportunity cost, however, banks are indifferent between holding excess reserves and lending in the federal funds market. In fact, this has been the case since November 2008, when the target band and the effective federal funds rate fell short of the interest rate paid on reserve balances, and excess reserves ceased to indicate a misallocation of liquidity. Wu (2011) provides empirical evidence that the TAF significantly lowered banks’ liquidity concerns. Carpenter et al. (2013), in turn, find that this reduction in liquidity risk attenuated the drop in C&I bank loans after the bankruptcy of Lehman Brothers by up to 23%.28

Why are banks still holding large reserve balances? Although the opportunity cost of excess reserves, \( r^B_t - r^R_t \), has been zero or negative, the net cost of central bank credit, \( r^X_t - r^R_t \), has been strictly positive throughout the crisis. Does this mean that banks are loosing money on a substantial fraction of assets? Figure 4 shows that the high current level of reserves is due to the Fed’s asset purchase programs rather than its lending programs, which started to wind down in early 2009. The former imply a different cost-benefit calculation than the latter. In particular, the 3-month treasury bill rate has been below 25 bps, the interest rate paid on reserve balances, since October 2008. Accordingly, the net cost of holding excess reserves added through the Fed’s asset purchase programs might even be negative.

28 Evidence on the effectiveness of the Fed’s liquidity facilities before October 2008 is rather mixed. McAndrews et al. (2008) and Wu (2008) find that the overall effect of the TAF on the Libor-OIS spread amounted to about 57 and 31–44 bps, respectively, whereas Taylor and Williams (2009) find no evidence that the TAF relieved strains in the money market during the early stages of the financial crisis.
If the lending programs entailed efficiency costs to the banking sector, as assumed in the model, why didn’t the Fed use its asset purchase programs to provide liquidity at the peak of the crisis, instead? There are several advantages of the former over the latter. First, a broader range of banks could use the Fed’s lending programs, whereas asset transactions were conducted with a limited number of counterparties. Second, the cost of central bank credit served as an incentive device to promote targeted rather than broad lending. Third, exit from lending programs was automatic, by construction, whereas exit from asset purchase programs could put asset prices under renewed pressure. Hence, there was no need “to drain reserves” added through central bank credit or “to shore up the federal funds rate” using the rate paid on reserve balances (compare Bech and Klee, 2011), in order to contain inflationary pressure.

6 Conclusion

While the RBC model in this paper lacks many features of contemporary dynamic stochastic general equilibrium models, such as real and nominal rigidities or an explicit role for conventional monetary policy, it provides an unadulterated insight into how financial frictions can affect real economic activity and trigger a recession (compare Gertler and Kiyotaki, 2011). I find that acyclical limited federal funds market participation is irrelevant for the propagation of standard supply and demand shocks, as the aggregate bank balance sheet expands or contracts without an impact on the cost of financial intermediation. Yet, interbank borrowing and lending attenuates “uncertainty shocks”, i.e. changes in the variance of bank-specific deposit realizations. In terms of impulse responses, an exogenous reduction in federal funds market participation has very similar dynamic implications, except for the conditional comovement of excess reserves and interbank credit.

Unconventional monetary policy, modeled here as a liquidity injection into the banking sector subject to efficiency costs, represents an imperfect substitute for the federal funds market, which can attenuate the adverse effects of financial shocks. Although the central bank’s intervention might amplify the accumulation of excess reserves and crowd out federal funds, if implemented “agnostically”, the level of reserves does not reflect its effectiveness in promoting the supply of credit to the real economy.

Prior empirical research indicates that bank liquidity risk increased dramatically in August 2007, due to potential intraday payments for asset-backed commercial paper liquidity lines, whereas frictions in the federal funds market became important only in September 2008. After Lehman Brothers, federal funds transactions thus decreased on the extensive margin and increased on the intensive margin (see, e.g., Ashcraft et al., 2011; Afonso and Lagos, 2012). The Fed’s liquidity
facilities, in particular the newly created TAF, reduced banks’ liquidity risk and attenuated the
effects of the financial crisis on the U.S. economy, while also contributing to the striking patterns
in Figure 1. The theoretical model presented in this paper contributes to our understanding of
the empirical findings by unifying bank liquidity risk, limited federal funds market participation,
and unconventional monetary policy in an analytical tractable framework.

There are several directions for future research. First, the current model assumes that firms and
banks never default on their loans and that federal funds market participation is exogenous.
Assuming that firms, banks, or both are subject to endogenous balance sheet constraints, as
in Bernanke et al. (1999) and Gertler and Kiyotaki (2011), respectively, would add realism at
the cost of tractability. Second, deposit realizations in the model are i.i.d. over time. Although
this assumption is consistent with the non-persistence of bank-level reserve balances in the data
(see Ennis and Wolman, 2012), allowing for state heterogeneity between banks might challenge
Proposition 1 and affect the model’s aggregate dynamics. Finally, the theoretical predictions in
this paper ask for a thorough empirical investigation based on U.S. bank-level data along the
lines of Afonso and Lagos (2012) and Ennis and Wolman (2012).
Appendix A  Proofs

Proof of Proposition 2. Market clearing in the federal funds market corresponds to

\[ E_t [b_t(i)|d_t(i) \leq (\bar{\kappa} - 1)Z_t] + E_t [b_t(i)|d_t(i) > (\bar{\kappa} - 1)Z_t] = 0. \quad (A.1) \]

I.e., for a banking sector of unit mass, the conditional expectation of interbank borrowing left of the cutoff, \( \bar{\kappa}Z_t \), must equal the negative of the conditional expectation of interbank lending right of the cutoff.

Recall that \( f(d_t(i)) \) and \( F(d_t(i)) \) denote the pdf and cdf of bank \( i \)'s stochastic deposit realization in period \( t \) and \( d_t(i) \sim N(\bar{D}_t, \sigma_i^2) \). Derive first equation (25) from Case 1:

\[
E_t [b_t(i)|d_t(i) \leq (\bar{\kappa} - 1)Z_t] = \int_{-\infty}^{(\bar{\kappa} - 1)Z_t} b_t(i) dF(d_t(i)) \\
= \xi_t \int_{-\infty}^{(\bar{\kappa} - 1)Z_t} [(\bar{\kappa} - 1)Z_t - d_t(i)] dF(d_t(i)) \\
= \xi_t \int_{-\infty}^{(\bar{\kappa} - 1)Z_t} [(\bar{\kappa} - 1)Z_t - d_t(i)] f(d_t(i)) \cdot d(d_t(i)) \\
= \xi_t \left\{ (\bar{\kappa} - 1)Z_t \cdot F((\bar{\kappa} - 1)Z_t) - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\bar{\kappa} - 1} Z_t \cdot d(d_t(i) - D_t) \right\} \\
\quad \cdot \exp \left\{ -\frac{[d_t(i) - D_t]^2}{2\sigma^2} \right\} \\
= \xi_t \left\{ (\bar{\kappa} - 1)Z_t - D_t \right\} F((\bar{\kappa} - 1)Z_t) + \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{(\bar{\kappa} - 1)Z_t} \left\{ \exp \left\{ -\frac{[d_t(i) - D_t]^2}{2\sigma^2} \right\} \right\} \\
= \xi_t \left\{ (\bar{\kappa} - 1)Z_t - D_t \right\} F((\bar{\kappa} - 1)Z_t) + \frac{\sigma^2}{\sigma^2} \cdot f((\bar{\kappa} - 1)Z_t) \}
\]

Similarly, derive next equation (26) from Case 2:

\[
E_t [b_t(i)|d_t(i) > (\bar{\kappa} - 1)Z_t] = \int_{(\bar{\kappa} - 1)Z_t}^{\infty} b_t(i) dF(d_t(i)) \\
= \xi_t \int_{(\bar{\kappa} - 1)Z_t}^{\infty} [(\bar{\kappa} - 1)Z_t - d_t(i)] dF(d_t(i)) \\
= \xi_t \int_{(\bar{\kappa} - 1)Z_t}^{\infty} [(\bar{\kappa} - 1)Z_t - d_t(i)] f(d_t(i)) \cdot d(d_t(i)) \\
= \xi_t \left\{ (\bar{\kappa} - 1)Z_t \cdot [1 - F((\bar{\kappa} - 1)Z_t)] - \frac{1}{\sqrt{2\pi}} \int_{(\bar{\kappa} - 1)}^{\infty} \cdot d(d_t(i) - D_t) \right\} \\
\quad \cdot \exp \left\{ -\frac{[d_t(i) - D_t]^2}{2\sigma^2} \right\} \\
= \xi_t \left\{ (\bar{\kappa} - 1)Z_t - D_t \right\} \left\{ 1 - F((\bar{\kappa} - 1)Z_t) \right\} + \frac{\sigma}{\sqrt{2\pi}} \int_{(\bar{\kappa} - 1)Z_t}^{\infty} \left\{ \exp \left\{ -\frac{[d_t(i) - D_t]^2}{2\sigma^2} \right\} \right\} \\
= \xi_t \left\{ (\bar{\kappa} - 1)Z_t - D_t \right\} \left\{ [1 - F((\bar{\kappa} - 1)Z_t)] - \sigma_i^2 \cdot f((\bar{\kappa} - 1)Z_t) \right\} 
\]

The above results can be inserted into the market clearing condition (A.1) to obtain the equivalent of equation (27):
\[ \xi_t \left\{ [(\bar{k} - 1)Z_t - D_t] F((\bar{k} - 1)Z_t) + \sigma_i^2 f((\bar{k} - 1)Z_t) \right\} \\
\quad + \xi_t \left\{ [(\bar{k} - 1)Z_t - D_t] [1 - F((\bar{k} - 1)Z_t)] - \sigma_i^2 f((\bar{k} - 1)Z_t) \right\} = 0 \\
\iff [(\bar{k} - 1)Z_t - D_t] F((\bar{k} - 1)Z_t) + \sigma_i^2 f((\bar{k} - 1)Z_t) \\
\quad + [(\bar{k} - 1)Z_t - D_t] [1 - F((\bar{k} - 1)Z_t)] - \sigma_i^2 f((\bar{k} - 1)Z_t) = 0 \\
\iff [(\bar{k} - 1)Z_t - D_t] [F((\bar{k} - 1)Z_t) + 1 - F((\bar{k} - 1)Z_t)] = 0 \\
\iff (\bar{k} - 1)Z_t - D_t = 0 \quad (A.2) \]

Thus, for a symmetric probability distribution and a Calvo signal orthogonal to \( d_t(i) \), a necessary and sufficient condition for interbank market clearing is \((\bar{k} - 1)Z_t = D_t\). \(\blacksquare\)

**Proof of Corollary 1.** Note first that a banking sector of unit mass implies \( E_t l_t(i) = L_t \). From *Case 1* and *Case 2*,

\[ l_t(i) = \begin{cases} 
    d_t(i) + b_t(i) + Z_t & \text{if } d_t(i) \leq (\bar{k} - 1)Z_t \\
    \bar{k}Z_t & \text{if } d_t(i) > (\bar{k} - 1)Z_t 
\end{cases} \quad (A.3) \]

and furthermore from *Case 1*,

\[ l_t(i) = \begin{cases} 
    \bar{k}Z_t & \text{w. prob. } \xi_t \\
    d_t(i) + Z_t & \text{w. prob. } (1 - \xi_t) \\
    \bar{k}Z_t & \text{w. prob. } 1 
\end{cases} \quad (A.4) \]

Applying the probability weights of the three possible cases in (A.4), we can derive the ex ante expected lending to entrepreneurs by bank \( i \) as

\[ E_t l_t(i) = \xi_t \int_{-\infty}^{(\bar{k} - 1)Z_t} \bar{k}Z_t dF(d_t(i)) + (1 - \xi_t) \int_{(\bar{k} - 1)Z_t}^{(\bar{k} - 1)Z_t} [d_t(i) + Z_t] dF(d_t(i)) \\
\quad + \int_{(\bar{k} - 1)Z_t}^{\infty} \bar{k}Z_t dF(d_t(i)) \\
= \xi_t \bar{k}Z_t F((\bar{k} - 1)Z_t) + (1 - \xi_t) \left\{ \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{(\bar{k} - 1)Z_t} [d_t(i) - D_t] \cdot \exp \left\{ -\frac{[d_t(i) - D_t]^2}{2\sigma^2} \right\} d(d_t(i)) \\
\quad + D_t F((\bar{k} - 1)Z_t) \right\} + (1 - \xi_t) \bar{k}Z_t F((\bar{k} - 1)Z_t) + \bar{k}Z_t [1 - F((\bar{k} - 1)Z_t)] \\
= \xi_t \bar{k}Z_t F((\bar{k} - 1)Z_t) + (1 - \xi_t) \left\{ \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{(\bar{k} - 1)Z_t} \exp \left\{ -\frac{[d_t(i) - D_t]^2}{2\sigma^2} \right\} d(d_t(i)) \right\} \\
\quad + D_t F((\bar{k} - 1)Z_t) \\
\quad + \bar{k}Z_t - (1 - \xi_t) \bar{k}Z_t F((\bar{k} - 1)Z_t) \\
= \bar{k}Z_t - (1 - \xi_t) \bar{k}Z_t F((\bar{k} - 1)Z_t) + (1 - \xi_t) D_t F((\bar{k} - 1)Z_t) - \sigma_i^2 f((\bar{k} - 1)Z_t)(1 - \xi_t) \\
= \bar{k}Z_t - (1 - \xi_t) \left\{ [(\bar{k} - 1)Z_t - D_t] F((\bar{k} - 1)Z_t) + \sigma_i^2 f((\bar{k} - 1)Z_t) \right\}. \quad \blacksquare \]
Appendix B  The Model in Loglinear Form

Below,  \( \hat{x}_t \) denotes the percentage deviation of variable \( x \) from its stationary equilibrium in period \( t \), with the exception of the rates of return on financial assets, \( \hat{r}^D_t \), \( \hat{r}^Z_t \), \( \hat{r}^L_t \), and \( \hat{r}^R_t \), which are expressed in absolute terms, i.e. in percentage point deviations from steady state. Moreover, \( \hat{B}_t \) and \( \hat{R}_t \) are expressed in terms of deviations from their stationary equilibria in percent of steady-state deposits, \( D^* \). To simplify the notation, the superscript asterisk on all variables’ stationary equilibrium is omitted in the following.

Equations (B.1) to (B.4) correspond to the worker household’s FOCs as well as the corresponding intertemporal budget constraint. (B.5) to (B.7) are the banker household’s FOCs and the intertemporal budget constraint. (B.8) and (B.9) characterize the capital goods producer’s investment decision and the equation of motion of the capital stock, respectively. The loglinear production function and the entrepreneur’s FOCs are given by (B.10) to (B.13), while (B.14) determines the demand for bank loans. It is just equal to the productive capital stock, as the relative price of capital is constant.

\[
\begin{align*}
0 &= \hat{\lambda}^w_t + \hat{C}^w_t - \hat{\zeta}_t \quad \text{(B.1)} \\
0 &= \hat{\lambda}^b_t + \hat{w}_t - \gamma \hat{N}_t \quad \text{(B.2)} \\
0 &= \hat{\lambda}^w_t - E_t \left[ \hat{\lambda}^w_{t+1} \right] - \frac{1}{1 + r^D} \hat{r}^D_t \quad \text{(B.3)} \\
0 &= \hat{C}^w_t + D \hat{D}_t - w N \left( \hat{w}_t + \hat{N}_t \right) - (1 + r^D) D \hat{D}_{t-1} - D \hat{r}^D_{t-1} - \hat{\pi}^e_t \quad \text{(B.4)} \\
0 &= \hat{\lambda}^b_t + \hat{C}^b_t - \hat{\zeta}_t \quad \text{(B.5)} \\
0 &= \hat{\lambda}^b_t - E_t \left[ \hat{\lambda}^b_{t+1} \right] - \frac{1}{1 + r^Z} E_t \left[ \hat{r}^Z_{t+1} \right] \quad \text{(B.6)} \\
0 &= \hat{C}^b_t + Z \hat{Z}_t - (1 + r^Z) Z \hat{Z}_{t-1} - Z \hat{r}^Z_t - \hat{\pi}^b_t \quad \text{(B.7)} \\
0 &= \hat{q}_t \quad \text{(B.8)} \\
0 &= \hat{\kappa}_t - \delta \hat{K}_t - (1 - \delta) \hat{K}_{t-1} \quad \text{(B.9)} \\
0 &= \hat{Y}_t - \hat{A}_t - \alpha \hat{K}_{t-1} - (1 - \alpha) \hat{N}_t \quad \text{(B.10)} \\
0 &= \hat{Y}_t - \hat{w}_t - \hat{N}_t \quad \text{(B.11)} \\
0 &= \hat{Y}_t - \hat{r}^K_t - \hat{K}_{t-1} \quad \text{(B.12)} \\
0 &= \hat{r}^L_t - \hat{r}^K_t E_t \left[ \hat{r}^K_t \right] \quad \text{(B.13)} \\
0 &= \hat{L}_t - \hat{K}_t \quad \text{(B.14)}
\end{align*}
\]

(B.15) determines actual loan provision by the banking sector as a function of the pdf and cdf of idiosyncratic deposit realizations, while (B.16) corresponds to the aggregate balance sheet identity. (B.17) guarantees that the federal funds market clears in period \( t \), and (B.18) computes...
the aggregate level of interbank borrowing (= interbank lending). (B.19) and (B.20) are the loglinearized profit functions of goods-producing firms and banks, respectively, while (B.21) reflects the free-entry condition into financial intermediation. Recall that free entry implies that both expected individual and realized aggregate profits in the banking sector are driven down to zero.

\[
0 = LL_t - \tilde{\kappa}Z\tilde{Z}_t + (1 - \xi) \left\{ F((\tilde{\kappa} - 1)Z)[(\tilde{\kappa} - 1)Z\tilde{Z}_t - D\tilde{D}_t] + \sigma^2 f((\tilde{\kappa} - 1)Z)\tilde{\sigma}_t \right\} \\
- \left\{ [(\tilde{\kappa} - 1)Z - D] F((\tilde{\kappa} - 1)Z) + \sigma^2 f((\tilde{\kappa} - 1)Z) \right\} \xi_t \\
0 = D\tilde{R}_t + L\tilde{L}_t - D\tilde{D}_t - Z\tilde{Z}_t \\
0 = (\tilde{\kappa} - 1)Z\tilde{Z}_t - D\tilde{D}_t \\
0 = D\tilde{B}_t - \left\{ [(\tilde{\kappa} - 1)Z - D] F((\tilde{\kappa} - 1)Z) + \sigma^2 f((\tilde{\kappa} - 1)Z) \right\} \xi_t \\
- \xi \left\{ F((\tilde{\kappa} - 1)Z)[(\tilde{\kappa} - 1)Z\tilde{Z}_t - D\tilde{D}_t] + \sigma^2 f((\tilde{\kappa} - 1)Z) \right\} \tilde{\sigma}_t \\
0 = \tilde{\pi}_t - Y\tilde{Y}_t - q(1 - \delta)K(\tilde{q}_t + \tilde{K}_{t-1}) + wN(\tilde{w}_t + \tilde{N}_t) \\
+ (1 + r)qK(\tilde{q}_{t-1} + \tilde{K}_{t-1}) + qK\tilde{r}_t^L \\
0 = E_t\tilde{\pi}_t^{b} - (r^L - r^R)L\tilde{L}_t - L(\tilde{r}_t^L - \tilde{r}_t^R) + (r^D - r^R)D\tilde{D}_t + D(\tilde{r}_t^D - \tilde{r}_t^R) \\
+ (r^Z - r^R)Z\tilde{Z}_t + Z(E_t\tilde{r}_t^Z - \tilde{r}_t^R) \\
0 = \tilde{\pi}_t^b \\
\]

With unconventional monetary policy, equations (B.15), (B.16), (B.18), and (B.20) are adjusted accordingly. Moreover, (B.22) is the loglinear version of the feedback rule used for broad liquidity injections.\(^{29}\)

\[
0 = \tilde{X} - \phi(\tilde{r}_t^L - \tilde{r}_t^D) \\
\]

\(^{29}\)Recall that excess reserves \(\tilde{R}_t\), interbank borrowing \(\tilde{B}_t\), and central bank liquidity injections \(\tilde{X}_t\) are expressed in terms of percent deviations from steady-state deposits \(D\).
Appendix C  Probability Density Functions

Figure C.1: Effect of interbank borrowing & lending on aggregate bank loans to entrepreneurs

(a) Complete access to the federal funds market  
(b) Incomplete access to the federal funds market

Figure C.2: Effect of shocks to financial intermediation on aggregate volume of federal funds

(a) Uncertainty shock ($\sigma_t = 1 \to 1.1$) for $\xi_t = .5$  
(b) Financial friction shock ($\xi_t = .5 \to .4$) for $\sigma_t = 1$

Figure C.3: Effect of broad central bank liquidity injections in response to financial shocks

(a) Uncertainty shock ($\sigma_t = 1 \to 1.1$) for $\xi_t = .5$  
(b) Financial friction shock ($\xi_t = .5 \to .4$) for $\sigma_t = 1$
References


Tables

**Table 1:** Calibration of parameters for the stationary and dynamic simulation of quarterly data

<table>
<thead>
<tr>
<th>Benchmark Parameter Values</th>
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<tr>
<td>( \beta^w )</td>
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<td>( \beta^h )</td>
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<td>( \delta )</td>
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<td>( \sigma^* )</td>
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**Table 2:** Steady-state values corresponding to the benchmark parameter calibration in Table 1

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<td>( K^<em>/4Y^</em> )</td>
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<td>( C^<em>/Y^</em> )</td>
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<td>( C^{h*}/Y^* )</td>
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<td>( Z^<em>/L^</em> )</td>
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<tr>
<td>( B^<em>/D^</em> )</td>
<td>.0437</td>
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<tr>
<td>( R^<em>/D^</em> )</td>
<td>0</td>
</tr>
<tr>
<td>( D^<em>/4Y^</em> )</td>
<td>2.208</td>
</tr>
<tr>
<td>( L^<em>/4Y^</em> )</td>
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<tr>
<td>( \pi^{A^*} )</td>
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</table>

**Table 3:** Calibration of parameters for unconventional monetary policy interventions

<table>
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<tr>
<th>Monetary Policy Parameter Values</th>
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<tr>
<td>( \phi )</td>
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<tr>
<td>( r^{X^*} )</td>
<td>.00125</td>
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<td>( r^{R^*} )</td>
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Figures

Figure 1: Excess reserves and federal funds transactions of banks in the U.S. as a fraction of total deposits; weekly data from January 1975 to December 2011
Figure 2: Selected impulse responses to a shock in the variance of deposit realizations

**Vertical axes:** Deviations from steady state in % (percentage points for $r^L$ and $r^D$; as a fraction of steady-state deposits for $B$ and $R$); **Horizontal axes:** Quarters after the exogenous shock

Figure 3: Selected impulse responses to an increase in the federal funds market friction

**Vertical axes:** Deviations from steady state in % (percentage points for $r^L$ and $r^D$; as a fraction of steady-state deposits for $B$ and $R$); **Horizontal axes:** Quarters after the exogenous shock
Sources of reserve balances (weekly data)

**Figure 4:** Sources of reserve balances in the U.S. as a fraction of total deposits; weekly data from January 2007 to December 2012 (compare Ennis and Wolman, 2012, Figure 2)

**Notes:** Reserves added through Fed’s lending facilities sum “term auction credit”, “central bank liquidity swaps”, and “loans”, while Reserves added through asset purchases equal “total factors supplying reserve balances” less “total factors, other than reserve balances, absorbing reserve funds” and reserves added through lending facilities. All time series are downloaded from the H.4.1 Statistical Release of the Board of Governors of the Federal Reserve and expressed as a fraction of total deposits.

**Figure 5:** Selected impulse responses to an uncertainty shock with targeted liquidity injection

**Vertical axes:** Deviations from steady state in % (percentage points for $r^L$ and $r^D$; as a fraction of steady-state deposits for $B$ and $R$); **Horizontal axes:** Quarters after the exogenous shock
Figure 6: Selected impulse responses to an uncertainty shock with broad liquidity injection

**Vertical axes:** Deviations from steady state in % (percentage points for $r^L$ and $r^D$; as a fraction of steady-state deposits for $B$ and $R$); **Horizontal axes:** Quarters after the exogenous shock

Figure 7: Selected impulse responses to a financial friction shock with targeted liquidity injection

**Vertical axes:** Deviations from steady state in % (percentage points for $r^L$ and $r^D$; as a fraction of steady-state deposits for $B$ and $R$); **Horizontal axes:** Quarters after the exogenous shock
Figure 8: Selected impulse responses to a financial friction shock with broad liquidity injection

**Vertical axes:** Deviations from steady state in % (percentage points for $r^L$ and $r^D$; as a fraction of steady-state deposits for $B$ and $R$); **Horizontal axes:** Quarters after the exogenous shock