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Licensing with Free Entry

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Abstract

We introduce a fairly general licensing model with an endogenous industry structure – in terms of number of active firms – and general licensing contracts. We show that when the patentee can employ contracts that can condition on market entry or price, it can implement an outcome that yields monopoly profits by awarding the license to a single firm. Furthermore, when the patentee can only use contracts based on the quantities of the licensees, it still captures the entire market via a single licensee, albeit not at the monopoly price. Commonly assumed two-part tariff contracts cannot duplicate this last outcome and yield lower profits. We discuss the welfare implications of various contractual schemes.

Keywords: Patent licensing, free entry, quantity competition.
JEL Classification: D45, K11, L11, L13, L21, L41.

Introduction

A patent on an innovation provides the patentee with monopoly rights over it, but to what degree this monopoly power translates into profits depends, beside the magnitude of the technological improvement, on the patentee's efficiency in licensing it to other firms and the access to older technologies by outsiders. Moreover, licensing contracts often include relatively complex clauses, such as milestone payments, retroactive rebates, and most-favored-nation provisions (see Thursby et al. (2001) and Razgaitis (2007)). In this paper, we investigate the relationship between contractual terms and the resulting welfare effects and innovation incentives.

There is an extensive literature on the licensing of a new technology and most of the theoretical models are based on the framework established by Kamien and Tauman (1986) and Katz and Shapiro (1986). In this framework, there is a fixed number of firms using an old technology and an outside innovator that has acquired a patent for a technology that lowers costs. These studies consider a restricted set of contractual forms, starting with pure fixed-fee and pure per-unit fee contracts. More recently, revenue sharing and tariffs with multiple components, e.g. two-part tariffs, have been put in focus, for example, see San Martín and Saracho (2010). A typical result is that the new technology is fully diffused as the innovation is licensed to all firms that were previously active in the market.[1] In most cases the full industry surplus cannot be extracted by the innovator due to the inefficiency of the contractual terms employed.[2]

This paper extends the literature in a number of directions. First, we endogenize the number of active firms, which will depend on the terms of the contracts offered.[3] Moreover, we assume that any number of non-licensees can choose to operate in the market using the old technology. Secondly, we consider more general contracts, allowing for non-linear quantity and revenue based licensing fees. Starting from general contractual forms with almost no restrictions, we stepwise reduce the contractual space, thereby illustrating how the market structure and output depends on the contractual form used by the patentee. Finally, our model utilizes a general demand function and takes the integer constraint on the number of firms into account.

Our first result is that a patentee who can employ contracts conditional on the quantity and revenue of its licensees achieves the monopoly profit associated with the new technology by signing a *single* contract. The argument is that revenue and quantity based

[1] An excellent review of the early literature appears in Kamien (1992).

[2] Exceptions to this last point are Erutku and Richelle (2006) and Erutku and Richelle (2007).

[3] Tauman and Zhao (2018) consider a restricted entry setting, where new firms can only enter if they have access to the new technology.

incentives allow the patentee to implement a reaction of its licensee which is aggressive to entry while producing the monopoly quantity on the equilibrium path when no entry takes place.

Next, we consider contracts that only depend on the output of the licensees. With a single licensee, the patentee manages to deter the entry by all non-licensees, but monopoly profits are not attainable. Despite the fact that the licensee is effectively rendered a monopolist, it is necessary to commit to a high level of output in order to deter entry. If the patentee cannot discriminate among its licensees, it is never optimal to issue more than a single license with general quantity based contracts.

Commonly used more restrictive quantity based contracts, such as two-part tariff and fixed fee contracts, are not optimal in the class of quantity based contracts, as they perform worse than - for example - quantity forcing. Moreover, with fixed fee contracts accommodation of entry becomes possible. The basic intuition is that the patentee may find it optimal to let an inefficient independent producer enter the market as opposed to fighting off entry, where the latter may lead to lower prices. An additional finding is that with fixed fees or with two-part tariffs with non-negative royalties, the patentee will issue multiple licenses in equilibrium.

Only a single contract will be issued with all but two of the contractual forms we consider. Moreover, independently of the contracts, we predict that post-innovation the number of active firms is lower. This finding is in sharp contrast with the findings of Sen and Tauman (2007), Farrell and Shapiro (2008), and Giebe and Wolfstetter (2008) who show that in their setting (no fixed costs and two-part tariff contracts) almost all existing firms would receive a license. Similarly, Tauman and Zhao (2018) show in a setting with potential entrants that diffusion is typically wide, even including previously inactive firms, and may be limited only for 'sufficiently drastic' innovations.

Our results also differ from the findings of Erutku and Richelle (2006) and Erutku and Richelle (2007) who employ more general contracts and show that all firms receive a license. They show that for a fixed number of firms non-linear quantity based contracts allow the innovator to extract monopoly profits. Their contracts are designed such that all firms are licensees and in aggregate produce the monopoly quantity while no firm has an incentive to deviate. The presence of fixed costs and the endogenous market structure are the main reasons why our results differ. Consequently, we are able to explain how a non-drastic innovation can change the market structure drastically.

In our model, consumer welfare decreases in contractual complexity. Furthermore, among all contractual forms that lead to a single license being issued, the less complex the licensing contract, the higher social welfare tends to be. Finally, the investment incentive to reduce marginal costs is largest with fixed fee contracts and smallest with conditional contracts, as the gains from the cost reduction increase in the quantity produced.

Our results can be used to guide competition policy decisions on licensing contracts. Authorities face a trade-off: maintaining the incentives to innovate while curbing excessive market power that could arise from a patent. In light of our results, contracts that purely aim at reducing competition among licensees – e.g. retroactive rebates – should be viewed critically. This is also in line with the stated objective of the authorities in the US and EU. Although the US and EU licensing guidelines do not discuss specific contractual terms, it is clearly stated that licensing contracts "*which have as their object or effect the prevention, restriction or distortion of competition*"[4] are prohibited. In our paper we show that in the context of endogenous entry any licensing contract used by the innovator will have the objective and effect of eliminating or restricting competition, and thus might be viewed as anticompetitive. However, the welfare effects depend on the actual contractual terms and not only on the number of competitors in the market.

The paper is organized as follows. In Section 1, we outline our model. Section 2 presents our main results for different complexity of contracts. Following the findings on market outcomes, in Section 3 we provide our welfare results. Section 4 concludes.

1 Model

Consider a homogeneous product market with infinitely many identical firms that have access to the same production technology which allows them to produce at marginal cost c. In order to operate in the market, a firm has to incur a fixed cost $F > 0$. For our general results to hold, the fixed costs can be arbitrarily small, as long as they are positive.[5] All active firms compete in quantities, where q_i denotes the output choice of firm i. The industry inverse demand is given by $P(Q)$, where Q is the total output supplied and $P'(Q) < 0, P''(Q)Q + P'(Q) < 0$, i.e. inverse demand is decreasing in quantity and satisfies the condition of strategic substitutability.[6] An outside innovator, I, acquires a patent for a new production technology that lowers the marginal cost of production to $c - \delta$, where $c > \delta \geq 0$, while leaving the fixed cost unchanged. The innovator can license

[4]EU Commission Licensing Guidelines (2014)

[5]The existence of fixed costs even for incumbents in the industry has the realistic implication that firms which can only sell very low quantities do not stay in the market.

[6]These demand assumptions imply that, for all non-negative quantities, marginal revenue of any firm is decreasing in own quantity, as well as in the quantity of competitors. This implies negatively sloped best responses (strategic substitutability), if marginal costs are constant. However, licensing payments can possibly overturn this.

the technology to a number of other firms. In order to exclude the uninteresting case of a natural monopoly, or a blocked entry outcome, we are going to assume that fixed costs are sufficiently low such that when a firm using the new technology produces the monopoly output, there will be entry into the market. This assumption also implies that pre innovation there are at least two firms active in the market.

It is clear that what I and its licensees can achieve in terms of profits depends on the set of contracts that are available to them. We define a contract in terms of payments from the licensee to the innovator. We assume that contracts cannot be discriminatory, such that each licensee gets the same contract.[7] We look at contracts that specify payments $t(X)$, where X is a vector of contingencies. Initially, we allow X to contain all relevant contingencies that are observable by the contractual parties in the game. Later, we step by step restrict these contingencies.

In our setting, the innovator can offer a licensing contract to many identical firms. We assume that the innovator will use an auction mechanism that ensures that he gets the total profit of a licensee up to its outside option. In the auction, the innovator offers L identical licensing contracts, $t(X)$, for the new technology. The licenses are then allocated to the L highest bidders. The relevant outside option of a potential licensee is then the profit of a non-licensee given the number (typically one) of firms that have received a license. This outside option in general depends on the terms of the licensing contract. In most of the cases we analyze, the non-licensees will not be able to obtain positive profits when the intended contracts are accepted, and hence the outside option for potential licensees will be zero.[8] In the few cases where entry is accommodated and the entrant is able to make a positive profit, given the free entry nature of the market, we feel safe to assume that the expected outside option for any one firm is zero.

The timing of the game is as follows:

- *Stage 1:* The innovator auctions L observable and non-renegotiable licensing contracts.
- *Stage 2:* Having observed the distributed licensing contracts, the non-licensee firms decide whether to become active. All firms, including the licensees, that become active incur the fixed operating cost F.
- *Stage 3:* All active firms observe the number of active firms and then compete by choosing their quantities simultaneously and non-cooperatively.

[7] This assumption is not without loss of generality. However, both E.U. and U.S. guidelines on licensing of innovations make it clear that discriminatory practices will be scrutinized by the authorities.

[8] Given the equilibrium outcomes of the following subgames, the revenue equivalence theorem holds as all bidders have identical valuations, which are equal to zero, in the auction stage.

We analyse the subgame perfect equilibria of the game. We focus on contracts that yield unique equilibria in the subgame starting after the auction, in stage 2. This excludes contracts that make the licensee exactly indifferent between different actions. Also this excludes asymmetric equilibria between multiple licensees with identical licensing contracts. Note that this ensures that the valuations of the licensees in the auction are well defined. Also, the possibility of a coordination failure is eliminated.

2 Licensing contracts

2.1 Contracts that implement the maximum industry profit

The maximal industry profit is the natural upper bound for the profit the innovator can obtain. Let us first characterize the industry profit maximizing outcome and then find contracts that yield maximal industry profits to the innovator.

Industry profits depend on the set of active firms, who have to pay F, and the quantity choice of each active firm. The set of active firms consists of two subsets, firms with the new production technology and firms with the old production technology. Clearly, industry profits are maximized if total industry output is produced with the best available technology. Hence, defining N as the number of active firms with the new technology, maximal industry profits are defined by

$$\max_{N,\{q_i\}_{i=1}^N} \sum_{i=1}^{N} \pi\big(q_i, \sum_{i=1}^{N} q_i; c-\delta\big),$$

where $\pi(q_i, q_{-i}; c-\delta) \equiv [P(q_i+q_{-i}) - (c-\delta)]\, q_i - F$. Note that because of positive fixed costs, productive efficiency requires that only one firm is active. Hence, industry profits are maximized for $N = 1$. Moreover, this firm has to produce the monopoly quantity associated with the new marginal cost $c-\delta$, $q^m \equiv \arg\max_q \pi(q, 0)$.

This outcome can be achieved by auctioning off a single licensing contract. The contract has to provide incentives to produce the monopoly quantity in case no other firm has entered the market. Off the equilibrium path, when entry takes place, the contract has to provide incentives to the licensee to produce a sufficiently large quantity to deter entry. Define q^d as the smallest quantity such that it is not profitable for an additional firm to enter the market, i.e., q^d is the smallest quantity such that $\pi(q, q^d; c) < 0, \forall q$.[9]

One obvious example of an optimal contract is one which conditions the payment of the

[9] In the rest of the paper, in order to save on space we will drop the cost parameter from the notation of the profit function. In all cases below it should be clear what the corresponding marginal costs are.

contract on the quantities of the single licensee depending on the number of active firms. The power of the contract lies, first, in the commitment that it provides. By conditioning payment on contingencies, the contract allows a commitment to conditional actions, i.e., to make credible off-equilibrium threats without having to choose suboptimal actions on the equilibrium path. This feature naturally holds for the more realistic contractual forms we consider below. Note that monopoly profits can be obtained even if there is no cost advantage of the new technology, $\delta = 0$. With no cost advantage the market is monopolized purely through the vertical contract.

For various reasons, it is unlikely that a licensing contract can condition directly on the number of non-licensees in the market, e.g. entry is not easily verifiable, or such contracts might invite objections from antitrust authorities. We now show that it is possible to achieve the monopoly outcome with contracts that are only conditional on the quantity and the revenue of the licensee. This result resembles that of Erutku and Richelle (2007), however we demonstrate that a single licensee is sufficient to achieve monopoly profits while they require such contracts to be offered to all existing firms. By conditioning payments on revenues the contract indirectly conditions on market price, which in turn is affected by the number of active firms in the market. Conditioning payments on revenue is thus a perfect substitute for conditioning on the number of non-licensees in the market.

Formally, the contract specifies payments from each licensee to the patentee $t(q_i, R_i)$, where $R_i \equiv P(q_i + q_{-i})q_i$ is the revenue of the licensee. Without further restrictions on the set of contracts, we can establish the following result.

Proposition 1 *The innovator can get monopoly profits auctioning a single licensing contract with payments that are only conditional on the quantity and the revenue of the licensee.*

Proof. See the Appendix. ■

Proposition 1 reveals that contracts which can condition on the quantities of the non-licensees, and thus indirectly on the number of active firms, are sufficient to achieve the monopoly outcome. While this result relies on revenue-based payments, in general it is only necessary that the contract has sufficient instruments to condition on own quantity and the aggregate quantity of others. By the same argument, contracts that can more directly condition on the market price or (aggregate) quantity of non-licensees are outcome equivalent. A feature of these contracts is that the total payment for the deterrence quantity, q^d, is lower than that for the smaller monopoly quantity, q^m. In this respect, these contracts resemble so-called retroactive rebates, like – for example – an all-unit discount on per-unit payments if a certain quantity threshold is reached.

In many markets prices and revenues might not be easily observable or verifiable. In the next step we start imposing restrictions on the contracts, such that they can no longer be made directly conditional on market price, licensee revenue, or the aggregate quantity of the non-licensees.

2.2 Contracts conditional on own quantity of output

We now focus on contracts that can only base payments for a license on the quantity of output of the licensee. Special cases of such contracts include two-part tariffs, three-part tariffs, or quantity forcing agreements. We will show that given the limited set of contracts, the innovator can still foreclose the market. The monopoly profit is, however, not attainable.

Any contract offered by the patentee will lead to a set of quantities chosen by its licensees. A contract is defined by a payment function $t(q_i)$. This payment does not have to follow a specific functional form but is only contingent on the quantity of the licensee. Let us first derive a few results for general payment functions (i.e. general own quantity based contracts).

Consider for the moment that the patentee offers a single license. This turns out to be the optimal policy for the patentee. Multiple licenses are discussed subsequently. With payments that are contingent on the quantity sold by the licensee, the patentee is able to force a specific quantity. This is crucial because forcing a single quantity that the licensee produces irrespective of whether another firm is active allows the patentee to deter other firms from becoming active in the market. However, in equilibrium this implies an output greater than the monopoly quantity. In general, there could be two alternatives to forcing a single large quantity that deters outside entry: First, by giving the licensee incentives to reduce the equilibrium quantity if no other firm is active, but still maintaining that the licensee produces a deterring quantity if another firm becomes active. Secondly, by accommodating other firms in equilibrium.

Accommodating other firms is never optimal, since it would be more profitable to extend the quantity produced by the licensee, replacing the quantity of the entrants. Note that each non-licensee must have had non-negative profits as otherwise it would not have become active in the market.

Thus the only possibility to improve upon forcing the deterrence quantity would be to provide incentives to the licensee to produce a quantity $\hat{q}$ that is smaller than q^d, when no other firms becomes active.

The two incentive compatibility constraints, corresponding to the situations when the licensee is alone in the market and when another firm is active, respectively, are

$$\pi_i(\hat{q}, 0) - t(\hat{q}) \geq \pi_i(q^d, 0) - t(q^d)$$
$$\pi_i\left(q^d, q^e(q^d)\right) - t(q^d) \geq \pi\left(\hat{q}, q^e(q^d)\right) - t(\hat{q})$$

where $q^e(q^d)$ is the total best response output of the non-licensees when they expect the deterrence quantity to be produced.

Adding the two inequalities side-by-side and rearranging terms gives

$$\pi_i(\hat{q}, 0) - \pi_i(q^d, 0) \geq \pi_i\left(\hat{q}, q^e(q^d)\right) - \pi_i\left(q^d, q^e(q^d)\right) \tag{1}$$

which, given the assumption on market demand (strategic substitutability of quantities), is unsatisfied for any $q^e(q^d) > 0$.[10] Note that, for any larger quantity $q^{d'}$ such that $q^e(q^{d'}) = 0$, the condition (1) becomes an equality. This leads to the licensee being indifferent between choosing $q^{d'}$ and the – from the patentee's perspective – more profitable $\hat{q}$. Thus, the unique implementation fails.

In summary, with a single license the best the patentee can achieve is to produce the smallest deterrence quantity in equilibrium. To be complete, we also have to consider the case of multiple licenses. It turns out that the best that can be achieved with multiple licenses is still to produce a the deterrence quantity in equilibrium albeit with a lower industry profit due to duplication of the fixed costs.[11]

Proposition 2 *Focusing on symmetric equilibria among licensees, when contractual clauses can only be conditioned on own licensee quantity, the optimal licensing contract induces a single licensee to produce the minimal deterrence quantity* q^d.

Proof. See above discussion and the Appendix.

The intuition for the result is that it is impossible to overturn strategic substitutability, i.e. the incentive of the licensees to produce less in response to entry with just quantity based contracts. Not surprisingly, when contracts can only be conditioned on the quantities of the licensees, the market outcome is inferior for the patentee but better for consumers. How such restrictions on contractual form affect welfare and innovation incentives will be taken up in Section 3.

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[10] To see that condition 1 cannot hold, rewrite (1) as $-\int_{\hat{q}}^{q^d} \pi_i'(s, 0)ds > -\int_{\hat{q}}^{q^d} \pi_i'(s, q^e)ds$ and note that by strategic substitutes $\pi_i'(s, q^e) < \pi_i'(s, 0)$ for any $q^e > 0$, which yields a contradiction.

[11] Note that to prove this result we rely on the assumption of non-discriminatory licensing contracts. Absent this assumption we are unable to obtain definitive results. In the stylized examples that we considered, we observe that the patentee can improve upon this outcome using discriminatory contracts, however each time multiplicity of equilibria arises.

2.3 Two-part Tariff and Fixed Fee Contracts

We now turn our attention to a more restrictive set of contracts, namely the commonly used two-part tariffs and fixed fees. One obvious conjecture is that the patentee will not be able to achieve the monopoly profits resulting from more sophisticated contracts.

We first show that two-part tariffs perform strictly worse than general own quantity based contracts. The reason is that a two-part tariff contract that is designed to implement full deterrence has the property that the licensee produces strictly more than the minimal deterrence quantity in equilibrium when no other firm is active. Confirming the same logic, when the per-unit royalty is restricted to be non-negative, equilibrium profits will be lower. Finally, pure fixed fee contracts lead to the lowest gains for the patentee. A supplementary technical analysis to the arguments below can be found in the appendix.

■ **Two-part tariff contracts:** The contract specifies a two-part payment that includes a per-unit fee w and a fixed fee R.

First of all, it is no longer clear that entry accommodation is never optimal. Entry accommodation via letting one (or more) outside firm who has a higher marginal cost of production may keep the market price higher, and thus may be more profitable than entry deterrence, although with the latter the patentee receives all of the industry profit. However, it turns out that with two-part tariff contracts the patentee always chooses to deter entry. The reason is that with any contract that includes a per-unit fee (i.e. a royalty), the patentee can implement an off-equilibrium output of its licensees that is exactly equal to the minimum quantity needed to deter the entry of a non-licensee.

Lemma 1 *With two-part tariff licensing contracts, in equilibrium the patentee will license to a single firm to deter the entry of non-licensees.*

Proof. See the Appendix. ■

Next, let us show that the optimal two-part tariff contract will implement an equilibrium quantity which is greater than q^d. In order to deter the entry of non-licensees, the contract should implement the deterrence quantity q^d when entry occurs and a different quantity q^{TP} when there is no entry. These two quantities should satisfy the first-order conditions of profit maximization off (with non-licensees active) and on (only the licensee is active) the equilibrium path:

$$\begin{aligned} P'(q^d + q^e(q^d))q^d + P(q^d + q^e(q^d)) &= c - \delta + w \\ P'(q^{TP})q^{TP} + P(q^{TP}) &= c - \delta + w \end{aligned}$$

Subtracting the two equations side-by-side gives

$$\left[P'(q^d + q^e(q^d))q^d + P(q^d + q^e(q^d))\right] - \left[P'(q^{TP})q^{TP} + P(q^{TP})\right] = 0 \tag{2}$$

Equation (2) is not satisfied for any $q^{TP} < q^d$, as, by strategic substitutability of demand, the left hand side is strictly negative (see the Appendix for a detailed proof). With the help of a two-part tariff contract the patentee forecloses the market through a single license, however in equilibrium it must induce the production of a quantity that is larger than the entry deterrence quantity. Furthermore, the optimal two-part tariff includes a negative royalty, w, as with a non-drastic innovation the patentee can deter entry through a single license only by lowering the per unit costs of its licensee.

The above results are summarized in the following Lemma.

Lemma 2 *With two-part tariff licensing contracts,*

i) the optimal per-unit royalty is negative

ii) the equilibrium quantity is larger than the minimum quantity needed to deter entry.

Proof. See the above discussion. ■

■ **Two-part tariff contracts with non-negative per unit fees:** Consider the case where the per unit fee, w, is restricted to be non-negative. A series of papers including and not limited to Sen and Tauman (2007) impose this restriction with the caveat that antitrust or regulatory authorities frown upon, if not outright ban, negative royalties. This restriction is clearly detrimental to the patentee. Choosing a positive royalty raises the marginal costs of the licensees and makes it harder for them to deter entry, and therefore increases the number of licenses that need to be issued. Note that if one ignores the integer constraint on the number of licensees, the optimal two-part tariff in this case will be equivalent to a fixed fee payment as the optimal contract will set $w = 0$. Otherwise, the optimal two-part tariff will set w to a 'marginal' positive value such that the licensees do not produce an excessive amount with or without entry.[12] Since entry accommodation cannot be optimal[13], the patentee will distribute as many licenses such that, in the event an additional firm enters the market (with the old technology), the

[12]This could be another reason why upstream firms or patentees charge positive per-unit fees, even though double marginalization leads to lower profits. This explanation is more valid for industries where the number of (downstream) firms is small, such that the integer constraint is particularly relevant.

[13]Lemma 1 applies here as well, since the off-equilibrium quantity is exactly q^d.

sum of Cournot equilibrium outputs of the licensees is at least as large as the deterrence quantity. Formally, the optimal number of licenses L is given by

$$\sum_{i=1}^{L} q_i^C = q^d$$

where q_i^C is the Cournot equilibrium output of a licensee when an additional firm becomes active with a marginal cost of c. Note that given the number of licenses, there will be no incentive to enter the market. Without entry, every one of the licensees is going to produce a quantity which is larger than q_i^C and total output will exceed q^d. The fixed fees will then extract all of the operational profit. The number of licenses depends on the fixed costs of entry as well as the magnitude of the cost reduction δ. However, given that multiple licenses must be issued, it is straightforward to show that the equilibrium quantity under the non-negative royalty restriction is larger than that under the optimal two-part tariff.

■ **Fixed Fee contracts:** In the event the output of the licensees cannot be observed or contracted upon, fixed fee contracts are the only option. Since the patentee can extract all of the profit of its licensees using the fixed fee, it will choose the number of licenses to maximize their (gross) profits. The patentee might now be willing to sacrifice its monopoly position to achieve this goal.

If the patentee aims to deter the entry of non-licensee firms, it will have to distribute as many licenses such that, in the event an additional firm enters the market (with the old technology), the sum of equilibrium outputs of the licensees is at least as large as the deterrence quantity. Outside of specific parameter constellations, in the event of an entry by a non-licensee (off-equilibrium) the licensees will produce more than the necessary deterrence output.

Now imagine the alternative of accommodating the entry of one or more non-licensees by issuing one fewer license. Since the non-licensees have higher marginal costs of production, replacing a single licensee with a non-licensee will lead to a reduction in the total quantity produced on- and off the equilibrium paths. As such, the patentee will be able to implement a higher market price by allowing the entry of a less efficient firm. The trade-off is then between achieving larger industry profits (through accommodation) and having a larger (100%) market share (through entry deterrence).

Without putting additional restrictions on demand, it is difficult to make some comparative static analysis on the likelihood of accommodation. Instead, below we will provide a numerical example that shows its existence.

Example 1 *Let the demand function be given by $P(Q) = 19 - Q$ and let $c = 1, \delta = 0.75$, and $F = 1$. Given these values the quantity that deters the entry of a non-licensee becomes $q^d = 16$.*

1. *It takes the patentee $L = 10$ licenses to deter entry. In equilibrium the licensees produce a total output of 17.05 and the patentee earns $\Pi^d = 19.05$. When entry occurs, the licensees react by producing a total output of 16.25 units.*

2. *If the patentee issues only 9 licenses, exactly one non-licensee will enter the market. In equilibrium, the firms altogether produce 16.98 units of output and the patentee will earn $\Pi^a = 19.28$. When an* ***additional*** *non-licensee enters, the 9 licensees and the one non-licensee produce a total output of 16.125 units.*

At this point a few remarks are in order. First, if one ignores the integer constraint on the number of firms, then entry accommodation can no longer be optimal, as the exact deterrence output q^d can be implemented through fixed fee licensing. We believe that in the presence of fixed costs and within the framework of licensing, this is not a very acceptable assumption. Second, entry accommodation of two or more non-licensees can be optimal if the old technology used by them is much less efficient.

Since multiple licenses must be issued and the off-equilibrium output exceeds q^d, the equilibrium quantity under fixed fee contracts is larger than that under the optimal two-part tariff.[14] Given the additional fixed costs that have to be covered, it is clear that fixed fee contracts are strictly less desirable for the patentee than two-part tariff contracts.

Lemma 3 *With fixed fee licensing contracts,*

i) the patentee will sell $L \geq 1$ licenses, entry of non-licensees may be possible.

ii) the equilibrium quantity is larger than that with two-part tariff contracts.

Proof. See the Appendix. ■

The results of this section are summarized in the following proposition.

Proposition 3 *There is a clear ordering of the above considered contracts from the perspective of the patentee:*

i) With two-part tariff contracts, the patentee will license to a single firm that produces a quantity ***greater*** *than the deterrence quantity q^d in equilibrium, and entry of non-licensees does not take place.*

[14] The only possibility of $L = 1$ is under the accommodation of non-licensee entry. In that case, the equilibrium quantity is still larger than that with two-part tariffs.

ii) With two-part tariff contracts that involve a positive per-unit fee, the patentee will license to multiple firms where the aggregate equilibrium quantity is ***greater*** *than* q^d*, and entry of non-licensees does not take place.*

iii) The patentee earns strictly more when the per-unit fee is allowed to be negative, i.e. optimal two-part tariff contracts are strictly superior to those with a positive per-unit fee, which are weakly better than pure fixed fee contracts.

Proof. See the above discussion and the Appendix. ■

Proposition 3 provides the last step in confirming our earlier conjecture that the more restrictive the licensing contracts, the lower is the profit of the patentee. Fixed fee contracts perform worse than two-part tariff contracts, which in turn are worse than general own quantity based contracts for the patentee. We conclude by saying that the second and third parts of Proposition 3 provide a relative common ground between our paper and those in the literature that predict a full dispersion of the new technology. When the permitted licensing contracts are sufficiently restrictive, then the patentee will license its technology to more than a single firm. However, in this case the monopoly profit cannot be attained.

3 Welfare Comparisons

We first compare the market outcomes from the preceding sections according to consumer and aggregate welfare criteria, and then discuss innovation incentives. The trade-off concerning static welfare can be summarized like this: While the availability of more complete or sophisticated contracts enables the patentee to achieve market dominance and a monopoly position, the existence of entry costs and a non-decreasing returns to scale production technology points to efficiency gains from market concentration. However, given that outside of two exceptions the optimal number of licenses is one, it is possible to rank different contractual forms in terms of their social welfare properties. Furthermore, the monotone relationship between contractual complexity and aggregate equilibrium output makes it easy to compare contracts according to their effects on consumer welfare.

To summarize our findings, aggregate equilibrium quantities are ordered from largest to smallest as follows: fixed fee contracts, two-part tariffs, general own quantity based contracts (quantity forcing), revenue and own quantity based contracts. Industry profits naturally follow the reverse order; the larger the equilibrium quantity the lower is the profit of the patentee. Multiple licensees are issued only when contracts are restricted to fixed fee payments or two-part tariffs with non-negative royalties. Finally, entry of non-licensees occurs only under fixed fee contracts. The welfare implications of these findings are outlined in the next proposition.

Proposition 4 *Welfare properties of licensing contracts:*

i) Consumer welfare is highest with fixed fee contracts and lowest with contracts based on revenue and own quantity. In general, the more restricted the contractual space, the better off consumers are.

ii) Among all contractual forms that lead to a single license being issued, two-part tariffs lead to highest total welfare and contracts based on revenue and own quantity to the lowest total welfare. In general, the more restricted the contractual space, the higher total welfare.

Proof. See the above discussion. ■

It should come as no surprise that contracts that can condition on a larger set of market variables perform more poorly in terms of welfare, as the patentee is able to implement market outcomes that are closer to a monopoly. Still, licensing can improve welfare compared to the pre-innovation situation, where multiple firms will be active using the old technology.

The pre-innovation equilibrium number of active firms, N, and aggregate equilibrium output will be determined by

$$\left[P(Nq^N) - c\right] q^N > F \geq \left[P\left((N+1)q^{N+1}\right) - c\right] q^{N+1} \tag{3}$$

where $q^N(q^{N+1})$ is the output of a single firm in a symmetric Cournot equilibrium with $N(N+1)$ firms with marginal costs of c. From the definition of the deterrence quantity, q^d, it is clear that $q^d \leq Nq^{N+1} < Nq^N$. The deterrence quantity is chosen to set the profit of an extra firm to *exactly* zero, and is therefore less than or equal to Nq^{N+1}. Given our assumptions on demand, q^{N+1} must be less than q^N, hence the second inequality follows. This implies that when available contracts are not restricted to two-part tariffs or fixed fees, consumers will be hurt from (the licensing of) the innovation, as the market is monopolized and the equilibrium price rises. Total welfare may still very well improve as all of the output will be now produced at lower costs and the industry will save on fixed costs. Without further restrictions on the parameters of our model, more concrete conclusions cannot be reached.

Our welfare discussion has so far focused on static efficiency. The usual trade-off for dynamic efficiency is between providing incentives to innovate through a patent that grants monopoly rights versus a distortion in allocative efficiency that results from the monopoly. Let us briefly argue why this trade-off might be different in this context. Suppose that there is an additional initial stage to our model, in which the innovator decides how much to invest in researching the new technology. Higher investment deterministically increases δ, the cost reduction of the new technology. In particular, $\zeta(\delta)$ measures the development cost of the cost reduction. Assume that ζ is twice continuously differentiable and weakly convex, with $\zeta(0) = 0$. Within our model, patenting the new technology gives the innovator the possibility to write licensing contracts, which in turn gives him the power to monopolize the market. However, the incentives to raise the magnitude of cost reduction do not increase when the market power from the contract increases. In fact, the incentive to invest in a cost reduction only depends on the equilibrium quantity.

Proposition 5 *Investment incentives and the value of a patent do not coincide.*

i) The value of the patent increases in the contractual freedom and is largest if contracts can be based on revenue and own quantity.

ii) The patentee invests most into increasing δ if it can only use fixed fee contracts and least if it can use contracts based on revenue and own quantity. Investment is always below the socially optimal level.

Proof. See the Appendix. ■

Interestingly the value of the patent does not only consist of the value of the cost reduction but additionally of the value of the commitment achieved through the licensing contract. While the commitment value increases if the contract can be directly or indirectly conditioned on entry, the value of the cost reduction increases in the quantity and, thus, decreases in the level of commitment. In the flavor of the replacement effect, as stated by Arrow (1962), market power reduces the incentive to invest in a cost reduction.

4 Conclusion

In this paper we extend the literature on the licensing of a cost reducing technology in two dimensions. First, we endogenize the market structure by considering an explicit decision by each firm to become active in the market. In this respect, the pre- and post-innovation number of firms are endogenously determined. Second, we allow for more general contractual terms which have not been considered in the previous studies.

The ad hoc limiting of the number of firms in a market, as we show, can lead to sharply different results than when no such assumption is made. Our model is also general in the sense that we refrain from putting any parametric restrictions on market demand.

In the context of a cost reducing innovation and homogeneous goods competition our paper arrives at two results: First, when the patentee can employ contracts that can condition on certain market variables such as entry or price, it will achieve the maximum industry profit by licensing its innovation to a single firm. Second, when the patentee can only employ contracts based on the quantities of its licensees, then it will deter the entry of all other firms by enforcing a high enough quantity on a single licensee. However, maximum monopoly profits will no longer be attainable. Multiple licenses will only be issued when negative per-unit fees, or more generally speaking, negatively sloped payment functions are excluded.

From a social welfare point of view it is not easy to dismiss more sophisticated, conditional contracts as harmful. Although more sophisticated contracts lead to a reduction in total output, there are efficiency gains to take into account. Consumer welfare monotonically decreases in the complexity of contracts. In contrast, the value of the patent for the innovator rises in the complexity of available contracts.

The predicted extreme market outcomes contradict the casual empirical observations that not all innovations result in a monopoly market structure and that not all technologies are exclusively licensed. The finding we present calls the most common assumption of the papers in the literature in question: full contractual commitment.

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5 Appendix

Proof of Proposition 1

As a first step, note that a contract that has payments $t(q_i, R_i)$ is equivalent to a contract that has payments $\tau(q_i, q_{-i})$. To see that, note that $R_i(q_i, q_{-i})$ is strictly monotone in q_{-i} as $P(q_i, q_{-i})$ is strictly decreasing in q_{-i}. Thus R_i is a unique function of q_{-i} for any given q_i. Hence, by additionally conditioning payments on revenue the innovator can condition the payments indirectly on the aggregate quantity of non-licensees.

Suppose the innovator wants the licensee to pick the monopoly quantity if no other firm entered, but to choose a larger deterrence quantity if there is entry. As q^d is the smallest quantity to deter entry, the reaction of a non-licensee to q^d if it entered is a strictly positive quantity $q^e(q^d) > 0$. This is due to the strictly positive fixed operating costs. The incentive problem that ensures a unique equilibrium is the following: first q^m on the equilibrium path, when there is no non-licensee ($q^e = 0$):

$$\pi(q^m, 0) - \tau(q^m, 0) > \pi(q^d, 0) - \tau(q^d, 0), \tag{4}$$

and second, q^d off the equilibrium path.

$$\pi(q^d, q^e(q^d)) - \tau(q^d, q^e(q^d)) > \pi(q^m, q^e(q^d)) - \tau(q^m, q^e(q^d)).^{15} \tag{5}$$

From inspecting (4) and (5) it is clear that both incentive constraints can be satisfied independently from each other by conditioning the payment on q^e. Each individual incentive constraint can easily be satisfied by choosing sufficiently large transfers $\tau(q^d, 0)$ and $\tau(q^m, q^e(q^d))$. Note that it is sufficient to take only the incentives for q^m and q^d into account, as it is trivially possible to make all other quantity choices not attractive by specifying large payments otherwise. The innovator can extract the whole profit in the auction, because the outside option to the license is zero.

Proof of Proposition 2

Let us consider now if the patentee would be better off by offering multiple licenses. Suppose that the patentee has offered multiple licences, $L \geq 2$. By the assumption of

[15] The incentive constraints would be satisfied with strict equality for $q^e(q^d) = 0$. In such a case the licensee finds itself in a position where, for the purposes of optimal choice of its quantity, the situation with entry is indistinguishable from that with no entry. This makes the licensee obviously indifferent between choosing q^d and the – from the patentee's perspective – more profitable q^m. There is no contractual modification that the patentee can implement to make its licensee choose q^m over q^d. By our focus on unique equilibria we disregard such contracts.

non-discriminatory licenses, the contract to each licensee has to be the same. In general, symmetric as well as asymmetric equilibria in the quantity competition subgame are possible. However, note that the licensees are symmetrical, having identical licensing contracts and costs. Hence, any asymmetric equilibrium among licensees can only exist in a context with multiple equilibria (at least up to the identity of the firms). For example, for any continuous payment function, a symmetric mixed strategy equilibrium exists, which cannot have higher pay-offs than the general symmetric equilibrium derived below. This multiplicity gives rise to a potential coordination failure among licensees. Consequently, focusing on uniqueness excludes contracts that allow for asymmetric equilibria among the licensees.

In the symmetric equilibrium each licensee produces an identical quantity which depends on the quantity produced by non-licensees, q^e. Denote the quantity produced by a licensee in the subgame in which no entry takes place by $\hat{q}$ and in case entry takes place licensees in aggregate produce q^d. We will show that it is impossible to implement $\hat{q} < q^d$.

Each licensee faces the following incentive constraints

$$\pi(\hat{q}, (L-1)\,\hat{q}) - t(\hat{q}) \geq \pi\Big(\frac{q^d}{L}, (L-1)\,\hat{q}\Big) - t\Big(\frac{q^d}{L}\Big)$$

$$\pi\Big(\frac{q^d}{L}, (L-1)\,\frac{q^d}{L} + q^e(q^d)\Big) - t(\frac{q^d}{L}) \geq \pi\Big(\hat{q}, (L-1)\,\frac{q^d}{L} + q^e(q^d)\Big) - t(\hat{q})$$

These two conditions can be rewritten as

$$t(\frac{q^d}{L}) - t(\hat{q}) \geq \int_{\hat{q}}^{q^d} \pi'(x, (L-1)\,\hat{q})dx$$

$$\int_{\hat{q}}^{q^d} \pi'(x, (L-1)\,q^d + q^e)dx \geq t(q^d) - t(\hat{q})$$

If $q^d > \hat{q}$, then $(L-1)\,q^d + q^e > (L-1)\,\hat{q}$, and by strategic substitutes we have $\pi'(x, (L-1)\,\hat{q}) > \pi'(x, (L-1)\,q^d + q^e)$, which directly implies a contradiction:

$$t(q^d) - t(\hat{q}) \geq \int_{\hat{q}}^{q^d} \pi'(x, (L-1)\,\hat{q})dx > \int_{\hat{q}}^{q^d} \pi'(x, (L-1)\,q^d + q^e) \geq t(q^d) - t(\hat{q}).$$

Hence, it not possible to specify payments such that $q^d > \hat{q}$.

If the patentee wants to deter entry, the best it can do is to implement a single quantity q^d that is always chosen independently of whether entry took place. Note that the alternative would be to produce a larger quantity on the equilibrium path, which is even further away from the industry profit maximizing quantity q^m (by the assumption of non-drastic innovation). Since it possible to produce q^d with a single firm, it is most cost efficient to issue a single license, as it saves on fixed costs.

Proof of Lemma 1

The proof consists of two parts: First, we show that the aggregate (output) reaction of two firms to a change in the output of a third firm is independent of their costs of production. In particular, the aggregate reaction of a licensee and a non-licensee to a change in the output of another entrant (non-licensee) is identical to the reaction of two licensees. This means that the equilibrium output with only two licensees will be equal to the equilibrium output with one licensee and a non-licensee (i.e. accommodation), when in each case the off-equilibrium quantity is equal to q^d. As a result, the patentee cannot achieve a higher profit by offering a single two-part tariff license while accommodating the entry of one non-licensee than when it offers two two-part tariff contracts and deters the entry of additional firms. Second, we show that the optimal number of licenses with two-part tariff contracts equals one, when the patentee wishes to deter the entry of non-licensees. Together these two findings prove that with two-part tariffs the optimal outcome for the patentee is to achieve entry deterrence through a single license.

Part i) Consider the first order conditions of profit maximization for two firms with potentially different (but both constant) marginal costs, c_1 and c_2, that compete with an "outsider" third firm, firm O.

$$P'(q_1 + q_2 + q_O)q_1 + P(q_1 + q_2 + q_O) = c_1$$
$$P'(q_1 + q_2 + q_O)q_2 + P(q_1 + q_2 + q_O) = c_2$$

Adding the equations side by side and denoting the total output by these two firms $q = q_1 + q_2$, we get

$$P'(q + q_O)q + 2P(q_1 + q_2 + q_O) = c_1 + c_2$$

Using the implicit function theorem in order to find the change in q with respect to a change in q_O, we get:

$$\frac{dq}{dq_O} = \frac{P''(q + q_O)q + 2P'(q + q_O)}{P''(q + q_O)q + 4P'(q + q_O)}$$

which is independent of the marginal costs of production **and** the individual quantities. So if two firms are producing the deterrence quantity q^d together, as the third firm exits the market, the change in total output is independent of individual costs and quantities.

Part ii) Suppose L licenses are issued to deter the entry of non-licensees. The quantities chosen by a single licensee are defined by the first-order-conditions, on and off the equilibrium path respectively:

$$P'(q^D + q^e(q^D))\frac{q^D}{L} + P(q^D + q^e(q^D)) = c - \delta + w,$$
$$P'(q^*)\frac{q^*}{L} + P(q^*) = c - \delta + w.$$

Subtracting the equations side-by-side yields

$$P'(q^D + q^e(q^D))\frac{q^D}{L} + P(q^D + q^e(q^D)) = P'(q^*)\frac{q^*}{L} + P(q^*)$$

After multiplying both sides by L, rearrange terms to obtain

$$\left[P'(q^D + q^e(q^D))q^D + P(q^D + q^e(q^D))\right] - \left[P'(q^*)q^* + P(q^*)\right] = (L-1)\left[P(q^*) - P(q^D + q^e(q^D))\right]. \tag{6}$$

First note that $q^* > q^D$. Suppose to the contrary that $q^D > q^*$. The LHS is the difference between two marginal revenues, for a firm producing q^D and q^* respectively. Each marginal revenue is decreasing in own quantity and the quantity of other firms, which implies that the LHS is negative. Hence, the RHS would have to be negative as well, which is a contradiction as price decreases in quantity.

To observe that the equilibrium quantity increases in L inspect Equation 6. Recall that $q^D \leq q^* \leq q^D + q^e(q^D)$, which implies that $P(q^*) \geq P(q^D + q^e(q^D))$. For any L, consider the equilibrium quantities $q^*(L)$ and $q^*(L+1)$, with one more license sold.
For a proof by contradiction, suppose $q^*(L+1) \leq q^*(L)$. This implies that the RHS of Equation 6 is larger in case more licenses are sold. Thus the LHS would have to be larger as well, which is a contradiction as only the second term is affected and it has a negative sign. Hence, $q^*(L)$ is increasing in L and the optimal number of licenses is one.

Proof of Proposition 3

i) With two-part tariffs the innovator is able to extract the total industry profits and chooses the number of licenses that maximizes industry profits. Profits are maximized by implementing an equilibrium quantity closest to q^m. We know from Lemma 2 that the equilibrium quantity is larger than q^d and increases in L. To maximize industry profits the innovator implements the smallest quantity possible than ensures that no non-licensee becomes active by choosing the smallest $L = 1$.

ii) In order to implement the deterrence outcome with only fixed fee contracts, the patentee will have to issue multiple licenses as explained in the text. The licensees will become identical Cournot competitors with marginal costs of $c-\delta$. Since the minimum output level to deter entry is q^d, the number of licensees is the smallest L that satisfies

$$\sum_{i=1}^{L} q_i^C \geq q^d$$

where q_i^C is the Cournot equilibrium output of a licensee when an additional firm enters the market at a marginal cost of c. Formally, q_i^C is defined by the first order condition

$$P'(Lq_i^C + q^e(Lq_i^C))q_i^C + P(Lq_i^C + q^e(Lq_i^C)) = c - \delta.$$

Without entry we have $q^e = 0$ and the above condition will be satisfied for a $q_i^F > q_i^C$. Thus, fixed fee only contracts implement an output greater than q^d in equilibrium (without entry).

iii) For each one of the L licensees, on- and off-equilibrium marginal revenues must be equal (to $c-\delta$). Thus, for each licensee we have

$$P'(Lq^F)q^F + P(Lq^F) = P'(Lq^C + q^e(Lq^C))q^C + P(Lq^C + q^e(Lq^C))$$

where $q^e(Lq^C)$ denotes the output of the non-licensees when L licensees produce (off-equilibrium) quantities of q^C each. Note that we dropped the i subscript for the licensees, given that with identical fixed fee only contracts they will all produce equal quantities on and off the equilibrium path.

Multiplying both sides by L (summing the above condition over all L licensees) and rearranging terms we get

$$\begin{aligned}\left[P'(Lq^F)Lq^F + P(Lq^F)\right] - \left[P'(Lq^C + q^e(Lq^C))Lq^C + P(Lq^C + q^e(Lq^C))\right] = \\ (L-1)\left[P(Lq^C + q^e(Lq^C)) - P(Lq^F)\right] \qquad (7)\end{aligned}$$

Given that total industry output is greater off the equilibrium path than on it (i.e. $Lq^C + q^e(Lq^C) > Lq^F$), both sides of Equation (7) are negative valued. It follows then that

$$P'(Lq^F)Lq^F + P(Lq^F) < P'(Lq^C + q^e(Lq^C))Lq^C + P(Lq^C + q^e(Lq^C))$$

Since $Lq^C > q^d$, due to strategic substitutability of quantities we have $Lq^C + q^e(Lq^C) > q^d + q^e(q^d)$. In other words, total industry output is greater when L licensees with fixed fee contracts are deterring entry than when a single licensee with a two-part tariff is fighting off entry. Thus,

$$P'(Lq^F)Lq^F + P(Lq^F) < P'(q^d + q^e(q^d))q^d + P(q^d + q^e(q^d))$$

Comparing the above inequality with Equation (2) from the text, we have

$$P'(Lq^F)Lq^F + P(Lq^F) < P'(q^{TP})q^{TP} + P(q^{TP}).$$

So, marginal revenue for a monopolist producing Lq^F is smaller than its marginal revenue when it produces q^{TP} units. Given our assumptions on the demand function, it follows that $Lq^F > q^{TP}$. The equilibrium quantity produced by the licensees with fixed fee contracts is greater than the equilibrium quantity produced by a single licensee with a two-part tariff.

Proof of Proposition 5

i) The value of the patent equals the profit of the patentee from licensing the innovation. Without the patented innovation the innovator has zero profits as there is free entry. The revenue that can be extracted by the patentee with a license depends on the cost reduction δ and on the contractual form. Clearly, the less restricted the contracts are, the larger the licensing revenue and the larger the value of the patent.

ii) In cases with a single license, the patentee in the initial stage maximizes $(p(q) - c + \delta)q - \zeta(\delta) - F$. Here q is the equilibrium quantity of the continuation game, which depends on the available licensing contract. The optimal level of δ for the patentee is determined by the first-order condition (with respect to δ) to the maximization problem: $[p'q + p(q) - c + \delta]\frac{\partial q}{\partial \delta} + q = \zeta'(\delta)$.

   First, note that at $q = q^m$ the left hand side of the first-order condition equals q^m, since $[p'q + p(q) - c + \delta]|_{q=q^m} = 0$. This follows from an envelope theorem argument; marginal profit must be zero at the profit maximizing (monopoly) quantity.

   Secondly note that at $q = q^d$ the left hand side equals q^d as $\frac{\partial q}{\partial \delta}|_{q=q^d} = 0$, such that the first term is again zero. This is because q^d is defined through the zero-profit condition of a non-licensee and thus is independent of δ.

   Finally, consider the equilibrium output under two-part tariffs and fixed fee contracts. With two-part tariffs, it is clear from Equation (2) that the equilibrium quantity q^{TP} is independent of δ and we have $\frac{\partial q}{\partial \delta}|_{q=q^{TP}} = 0$, such that the first term in the first-order condition with respect to δ is once again zero. With fixed fee contracts, in the event of entry the patentee cannot always implement the exact deterrence quantity q^d (but instead a quantity at least as big as q^d), since the number of licenses has to be integer. Ignoring the integer constraint by assuming that $\sum_{i=1}^{L} q_i^C = q^d$, where q^C is again the Cournot equilibrium output of a licensee when an additional firm enters the market with a marginal cost of c. In this case, similar to the argument for two-part tariffs the equilibrium output produced by all licensees, Lq^F, will also be independent of δ.

To sum up, since we know that $q^m < q^d < q^{TP} < Lq^F$ and since ζ is weakly convex, the optimal δ^* strictly increases in the equilibrium quantity. Hence, $\delta^*(Lq^F) > \delta^*(q^{TP}) > \delta^*(q^d) > \delta^*(q^m)$.

Consider now the first-best situation. The social planner would let a single firm be active and set $p = c - \delta$. Then it would choose δ to maximize the welfare function

$$W(\delta) = \int_0^{p^{-1}(c-\delta)} [p(q) - c + \delta]\, \mathrm{d}q - \zeta(\delta) - F$$

The first-order condition is simply

$$p^{-1}(c - \delta) = \zeta'(\delta)$$

which implies that the optimal level of δ lets a single firm produce a quantity q^* such that 1) $p(q^*) = c - \delta$ and 2) $q^* = \zeta'(\delta)$. Clearly, q^* is greater than even Lq^F, and hence $\delta^*(q^*) > \delta^*(Lq^F)$.