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Spatial Competition with Capacity Constraints and Subcontracting

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Abstract

We characterize mixed-strategy equilibria when capacity constrained suppliers can charge location-based prices to different customers. We establish an equilibrium with prices that weakly increase in the costs to supply a customer. Despite prices above costs and excess capacities, each supplier exclusively serves its home market in equilibrium. Competition yields volatile market shares and an inefficient allocation of customers to firms. Even ex-post cross-supplies may restore efficiency only partly. We use our findings to discuss recent competition policy cases and provide hints for a more refined coordinated-effects analysis.

JEL classification: L11, L41, L61

Keywords: Bertrand-Edgeworth, capacity constraints, inefficient competition, spatial price discrimination, subcontracting, transport costs.

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1 Introduction

The well-known literature based on Bertrand (1883) and Edgeworth (1925) studies price competition in the case of capacity constraints – but does so mostly for homogeneous products and no spatial differentiation. A recent example is Acemoglu et al. (2009). We contribute at a methodological level with a model of spatial competition where capacity constrained firms are differentiated in their costs of serving different customers and can charge customer-specific prices. This leads to mixed price strategies with different prices for different customers, which is a new and arguably important addition to this strand of literature. One novel aspect of our analysis is that the mixed strategies induce cost-inefficient supply relations, such that transport costs are not minimized.

Various competition policy cases feature products with significant transport costs for which location or customer-based price discrimination is common. There are also merger control decisions in relation to such products that use standard Bertrand-Edgeworth models, which unfortunately do not take spatial differentiation and customer-specific pricing into account. For instance, in the assessment of the merger M.7009 HOLCIM/CEMEX WEST the European Commission argued "*that the most likely focal point for coordination in the cement markets under investigation would be customer allocation whereby competitors refrain from approaching rivals' customers with low prices.*" Moreover, it reasoned that "*given the low level of differentiation across firms and the existing overcapacities, it is difficult to explain the observed level of gross margins as being the result of competitive interaction between cement firms.*" As a supporting argument, the European Commission referred to a Bertrand-Edgeworth model with constant marginal costs and uniform pricing.[1]

Our model makes several predictions that can be related to the above reasoning: Even with overcapacities of 50%, we find that in a competitive equilibrium firms may always serve their closest customers ("home market"), and then at prices above the costs of the closest competitor. Firms set high prices in the home markets of rival firms, although a unilateral undercutting there seems rational in light of their overcapacities. Such a pattern is difficult to reconcile with previous models of competition. To answer the question of whether firms are indeed coordinating or competing, our model – which allows for spatial differentiation, location-specific pricing and capacity constraints at the same time – could therefore improve the reliability of competition policy assessments. In addition to the cement industry, the key features of our model, namely capacity constraints, a form of spatial differentiation, and price discrimination, can be found in a number of other industries like the production of commodities, chemicals, and building materials. The transportation costs in our model can also be interpreted as costs of adaption. For example, consulting firms may have expertise in a certain area but can serve demand in other areas with some additional effort. Moreover, market segmentation and the price discrimination of customers becomes more common in many consumer markets, due to new targeting technologies and increased potential for customer recognition.[2]

[1] See Section 9 for a more detailed discussion and references.

[2] See Villas-Boas (2004) and Esteves (2010) for price discrimination and Iyer et al. (2005) for targeted

We find that firms play mixed strategies in prices so that a firm sometimes serves a customer although another firm with lower costs has free capacity. This result of inefficient competition arises in a symmetric setting with efficient rationing, where firms have sufficient capacity to serve all customers and can perfectly price discriminate between customers. There thus seem to be enough instruments to ensure that prices reflect costs and the intensity of competition for each customer. There is also complete information about the parameters of the game, which means that the allocative inefficiency arises purely due to strategic uncertainty: As one competitor does not know which prices the other competitor will ultimately charge in equilibrium, the less efficient firm sometimes wins the customer. This natural insight that price competition can lead to strategic uncertainty and thereby inefficient outcomes is, to our knowledge, very rarely reflected in formal models.

The allocative inefficiency provides a rationale for cross-supplies between the competing suppliers. There is scope for such subcontracting when one supplier makes the most attractive offer to a customer, while another one has free capacity to serve that customer at lower costs. Cross-supplies can be observed in various industries. For instance, see Marion (2015) for a recent article on subcontracting in highway construction. We show that cross-supplies can lead to an efficient production in certain situations, but not in all. Firms refrain from subcontracting when this frees up the capacity of a constrained firm that has set low prices – as the additional capacity can increase competition on (otherwise) residual demand segments of the market.

The remainder of the article is structured as follows. We discuss the related literature in the next section, introduce the model in Section 3, study the case each firm has to charge a uniform price in Section 4, and the case that firms can price discriminate in Section 5. We compare the market outcomes with and without price discrimination in Section 6. In Section 7, we introduce subcontracting and in Section 8 we endogenize the capacities to demonstrate that excess capacity can occur in equilibrium when firms optimally choose their capacities and demand is uncertain. We conclude in Section 9 with further discussion on the inefficiency associated with competition, subcontracting, and Bertrand-Edgeworth arguments in competition policy.

2 Related literature

This article contributes to several strands of the existing literature. One strand addresses spatial price-discrimination. In a seminal article, Thisse and Vives (1988) investigate the choice of spatial price policies in the case of transport costs, but absent capacity constraints. They find that firms do not individually commit to mill pricing (which means that each customer pays the same free-on-board price plus its individual transport costs), but prefer to charge each customer prices according to the intensity of competition for that customer.[3] This results in a pattern of high prices for customers in the firms' home markets and low

advertising.

[3]This is essentially determined by the second most efficient firm's marginal cost for each customer.

prices for customers in-between the firms. We find that also with capacity constraints the equilibrium prices do not correspond to the simple mill pricing pattern. However, the pricing patterns with capacity constraints more closely follow the firms' own marginal costs and not only the intensity of competition.

We build on the classic literature based on Bertrand (1883) and Edgeworth (1925). This literature contains seminal contributions such as Levitan and Shubik (1972) who analyze price competition with capacity constraints, but with a focus on uniform prices and without (spatial) differentiation. There are a few articles and working papers which introduce differentiation in the context of capacity constrained price competition, notably Canoy (1996), Sinitsyn (2007), Somogyi (2013), and Boccard and Wauthy (2008). Canoy investigates the case of increasing marginal costs in a framework with differentiated products. However, he does not allow for customer-specific costs and customer-specific prices. Somogyi considers Bertrand-Edgeworth competition in the case of substantial horizontal product differentiation in a standard Hotelling setting. Boccard and Wauthy focus on less strong product differentiation in a similar Hotelling setting to Somogyi. Whereas Somogyi finds a pure-strategy equilibria for all capacity levels, Boccard and Wauthy show that pure-strategy equilibria exist for small and large overcapacities, but only mixed-strategy equilibria for intermediate capacity levels. For some of these models equilibria with mixed-price strategies over a finite support exist (Boccard and Wauthy, 2008; Sinitsyn, 2007; Somogyi, 2013). This appears to be due to the combination of uniform prices and demand functions which, given the specified form of customer heterogeneity, have interior local optima as best responses. Overall, these contributions appear to be mostly methodological and partly still preliminary.

In a related vein, there are mixed-strategy price equilibria in models with segmented customers, such as the model of sales by Varian (1980), and also customers with different preferences in Sinitsyn (2008, 2009). Based on this literature, it is conceivable that inefficiencies can arise if firms have asymmetric costs and charge uniform prices across different customer groups in a mixed strategy equilibrium. However, in this literature, pure strategy equilibria emerge if price discrimination is possible. In contrast, we show that allocative inefficiencies arise in a symmetric setting with efficient rationing and price discrimination.

In a follow-on project, we compare the outcomes of price competition and coordination using detailed billing data of cement sales in Germany (Hunold et al., 2018). Controlling for other potentially confounding factors, such as the number of production plants and demand, we show that the transport distances between suppliers and customers were, on average, significantly lower in cartel years than in non-cartel years. To develop the underlying hypotheses, we build on the present theoretical model and compare competition with collusion. We restrict attention to the case of uniform prices and consider a continuum of consumers to study how the allocative inefficiency varies in the degree of overcapacity.

The present article is also related to the literature on subcontracting relationships between competitors (also referred to as cross-supplies). With a subcontract, a firm can essentially use the production capabilities of a competitor. Efficiencies can, for instance, arise when a firm with decreasing returns to scale has won a large contract, so that subcontracting part of the production to an identical firm reduces costs (Kamien et al., 1989). Similarly, if there are increasing returns to scale, pooling the production can reduce costs (Baake et al., 1999). More indirectly, if firms with asymmetric costs compete, the resulting allocation of demand to each firm may not exactly minimize costs, such that again subcontracting increases efficiency (Spiegel, 1993).

The above literature on subcontracting has essentially pointed out a competitive effect, which depends on how the efficiency rents are shared between the two parties to the subcontract. If the receiver obtains the efficiency rent, its effective costs are lower as it uses the partly more efficient production technology of its competitor at costs. This tends to increase competition. Instead, if the cross-supplier obtains the efficiency rent, it forgoes a profit when competing as that reduces the probability of making rents with subcontracting. This tends to soften competition.[4] For instance, Spiegel (1993) points out that ex-ante agreements to cross-supply, which are concluded before firms compete, can dampen competition.

We contribute to this literature on subcontracting in several ways. We focus on ex-post subcontracting, which takes place after firms have competed in prices. First, we point out that horizontal subcontracting may also occur when firms are symmetric and there are no generic reasons for subcontracting. In particular, if there was a symmetric equilibrium in pure price strategies, there would be no scope for subcontracting. The only reason for subcontracting is that price competition with capacity constraints can lead to the allocations of customers to firms that do not correctly reflect the differences in production costs, although customer-specific pricing is feasible. We show that subcontracting can increase or decrease consumer surplus, depending on the distribution of the efficiency gains among the subcontracting competitors. Moreover, we show that – to our knowledge – there is a new reason for why firms may not engage in welfare-improving subcontracting. When a firm that produces at its capacity limit asks an unconstrained firm for a cross-supply to a customer which that firm can supply more efficiently, the unconstrained firm may deny this. The reason is that such a supply would leave the demanding firm with additional capacity, which can intensify the competition for other customers.

3 Model

Set-up

There are two symmetric firms. Firm L is located at the left end of a line, and firm R at the right end of this line. Four customers are located on the line, named 1, 2, 3, and 4 from left to right. The firms produce homogeneous goods, but differ in their costs of serving different customers. Each customer has unit demand and values the good at v. Firm L incurs costs of $1c$, $2c$, $3c$, and $4c$ to serve the customers from left to right. For firm R, there are costs of $4c$, $3c$, $2c$, $1c$ to serve the same customers. There are no other costs of supply. We assume that

[4] Marion (2015) finds that in California highway construction auctions the winning bid is uncorrelated with horizontal subcontracting and attributes this to an efficiency motive for cross-supplies. See also Huff (2012) for a similar study.

the valuation of the good is higher than the costs of serving even the most distant customer: $v > 4c$, so that each customer is contestable. See Figure 1 for an illustration. We mostly refer to these costs as transport costs in the analysis because we consider this to be most illustrative. Please keep in mind, however, that one can more generally interpret these as the firms' costs of serving different customers.

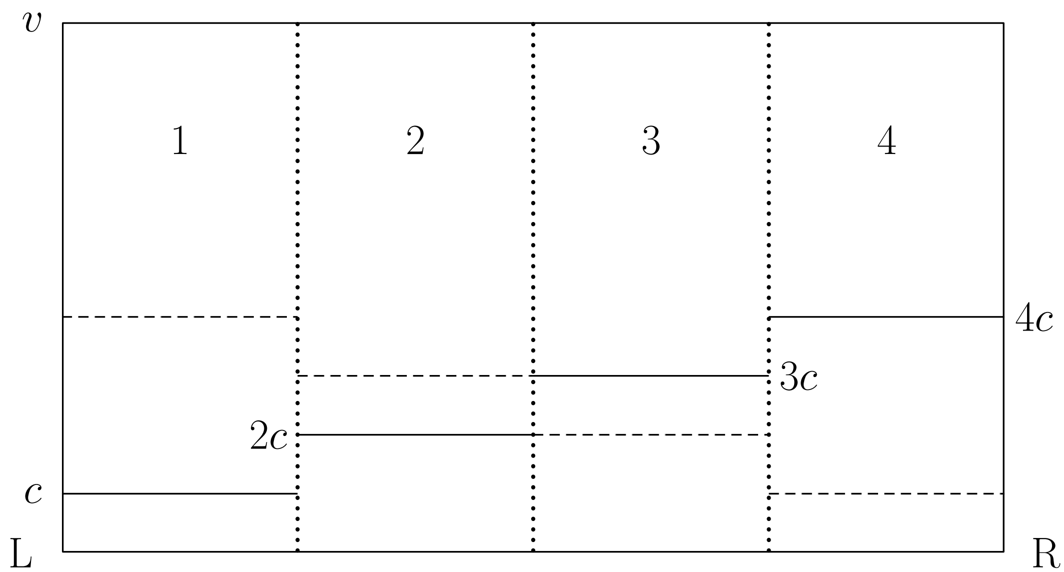

Figure 1: Customers 1 to 4 with unit demand and willingness to pay of v are located between firms L and R; the transport costs increase in the distance to each customer and range from c to $4c$.

Each firm $j \in \{L, R\}$ charges each customer $i \in \{1, 2, 3, 4\}$ a separate price p_i^j. A pure price strategy of firm j is a vector $(p_1^j, p_2^j, p_3^j, p_4^j) \in R^4$. In the case of a mixed strategy equilibrium, the strategy of firm j is a joint distribution of its prices: $F_j(p_1^j, p_2^j, p_3^j, p_4^j)$. We solve the game from the perspective of firm L and apply symmetry. If we suppress superscript j, p_i belongs to firm L.

The game has the following timing:

1. Firms L and R simultaneously set the eight prices p_i^L and p_i^R, $i \in \{1, 2, 3, 4\}$;

2. Customers are allocated to firms – according to prices and capacity constraints.

As a tie-breaking rule, we assume if both firms charge a customer the same prices, the customer buys from the firm with the lower transport costs. In our main case, each firm has the capacity to serve up to three of the four customers. Consequently, a single firm cannot serve the whole market, whereas overall there is 50% overcapacity.

We consider a small discrete number of customer segments to be an adequate approximation of certain real-world markets. Discrete customer segments could correspond to different cities or countries with different transport costs or regulations. This could result in different costs for the firms to serve these different markets. It is noteworthy that a small discrete number of customers also captures essential features of the case with a continuum of customers. Consider a continuous distribution of customers on a unit line. Still, the four customers would correspond to the market allocations that can arise if each firm charges a uniform price to all customers and has a capacity to cover three-quarters of the market.[5] We further discuss equilibria when there is a continuum of customers at the end of Subsection 5.3.

Rationing

We employ efficient rationing, in particular, we use the following rationing rule:

1. If one firm charges lower prices than the other firm to more customers than it has capacity, we assume that the customers are rationed so that consumer surplus is maximized. In other words, of those customers facing the lower price, the customer with the best outside option is rationed.

2. If the first point does not yield a unique allocation, the profit of the firm which has the binding capacity constraint is maximized (this essentially means cost minimization).

While this is not the only rationing rule possible, we consider this rule appropriate because:

- The employed rationing corresponds to efficient rationing (as, for instance, used by Kreps and Scheinkman, 1983) in that the customers with the highest willingness to pay are served first. A difference is, however, that the willingness to pay for the offers of one firm is endogenous in that it depends on the (higher) prices charged by the other firm. These may differ across customers, and so does the additional surplus for a customer from purchasing at the low-price firm.

- The rationing rule is geared towards achieving efficiencies, in particular for equilibria in which the firm's prices weakly increase in the costs of serving each customer. Our results of inefficiencies in the competitive equilibrium are thus particularly robust. For instance, in the case of proportional rationing each firm would serve even the most distant (and thus highest cost) customer.

- At least for the case of uniform prices ($p_1^j = p_2^j = p_3^j = p_4^j$), the same outcome is obtained if rationing maximizes the profitability of the firm with the low prices. This firm would also serve the closest three customers, as this minimizes the transport costs. In the case of proportional rationing, the main difference is that both firms serve all customers such that transport costs are higher and the profits are lower.

- The rationing rule is the natural outcome if the customers can coordinate their purchases: They reject the offer that yields the lowest customer surplus. This occurs, for

[5] For equal prices, each firm would serve half the market (two customers in the discrete case), whereas if one firm has a lower uniform price, it would serve three-quarters of the market (3 customers), and the other firm the residual demand of one-quarter (one customer).

instance, if interim contracts with side payments among the customers are allowed. It would also occur if there is only one customer with production plants at different locations.

- If a firm has to compensate a customer to which it made an offer that it cannot fulfill, this might also incentivize the firm to ration according to the customer's net utility from this contract.

In the following sections we solve the price game for Nash equilibria. Whenever firms are symmetric, we focus on symmetric equilibria. We study the game for two cases. We first analyze the case that firms cannot price discriminate between customers in Section 4 (this means $p_1^j = p_2^j = p_3^j = p_4^j$), and then study price discrimination in Section 5.

4 Equilibria under uniform pricing

In this section, we study the case that the firms cannot price discriminate, which means that each firm sets a uniform price for all customers ($p_1^j = p_2^j = p_3^j = p_4^j$). We first analyze the cases that each firm has either one or two units of capacity, which results in monopoly prices. We then turn to the case that each firm has four units of capacity, so that it can serve the whole market. This leads to highly competitive prices. We finally turn to the more complex case that each unit has three units of capacity, so that it can serve more than half of the market, but not the whole market. We show that this leads to an equilibrium with mixed price strategies when the willingness to pay is sufficiently high in relation to costs.

4.1 Scarce capacities of 1 or 2 units per firm

Suppose that each firm has the capacity to serve only one or two of the four customers. Consequently, it is an equilibrium in pure strategies that each firm sets its uniform price equal to the willingness to pay of v, and that each customer buys the good from the closest firm. The allocation of customers to firms follows from the rationing rule, which for equal prices allocates according to the cost level. The outcome is efficient as all customers are served by the firm with the lowest costs. Each firm obtains the highest profit that is feasible with its capacity. With one unit, the profit equals $v - c$; with two units, it equals $2v - 3c$. The consumer surplus is zero in each case. Total surplus is at the maximal level for the given capacities. For two units of capacity per firm, total surplus is $4v - 6c$. If there is one unit of capacity per firm, capacities do not suffice to cover the market. This implies that customers 2 and 3 are not served, which reduces total surplus to $2v - 2c$.

Note that there is no incentive of a firm to price discriminate across customers. This implies that this equilibrium persists when firms can price discriminate.

4.2 Abundant capacity of 4 units or more per firm

Suppose that each firm has capacity to serve all four customers. This means that the firms set prices without binding capacity constraints. It is thus an equilibrium in pure strategies that each firm sets the uniform price equal to its marginal cost of serving the third closest customer, $3c$, and that each customer buys the good from the closest firm.[6] Again, for symmetric uniform prices, customers are allocated to minimize costs. This is again efficient (for given capacities) in that all customers are served by the closest firm with the lowest transport costs. Each firm makes a margin of $3c - c = 2c$ from selling to the closest customer, and $3c - 2c = c$ from selling to the second closest customer. The equilibrium profit of a firm is thus $3c$. Consumer surplus is $4(v - 3c) = 4v - 12c$ and total surplus is at the maximal level of $4v - 6c$.

4.3 Limited excess capacity of 3 units per firm

Non-existence of a pure strategy equilibrium. Suppose each firm can serve at most three customers and both firms set prices as if there were no capacity constraints, as discussed in the previous subsection. Is this an equilibrium? As each firm charges a price of $3c$ to each customer, there is no incentive to lower the price as this would lead to additional sales below costs.[7]

In view of the other firm's capacity constraint, a now potentially profitable deviation is to charge all customers a price equal to the valuation v. All customers then prefer to buy from the other firm at the lower price of $3c$. However, as each firm only has the capacity to serve three customers, one customer ends up buying from the deviating firm at a price of v. Given the rationing rules, this is the customer closest to the deviating firm as this minimizes transport costs. The profit of the deviating firm is thus $v - c$. This is larger than the pure strategy candidate profit of $3c$ if $v/c > 4$.

The condition $v/c > 4$ for a profitable deviation is equivalent to the contestability assumption, such that no pure strategy equilibrium exists.

Mixed strategy equilibria. We now solve the price game for symmetric mixed strategy Nash equilibria. Such an equilibrium is defined by a symmetric pair of distribution functions. We now characterize an equilibrium in which each firm draws uniform prices (the same for all customers) from a single marginal price distribution F with support $[\underline{p}, v]$. There are no mass points in the marginal distributions of the prices. Let us now derive F to show that such an equilibrium exists.

When both firms only play uniform prices, each firm serves the nearest customer with certainty. To see this, note that if firm L charges the lowest (uniform) price, rationing implies

[6] There are equilibria with even lower prices. For instance, both firms charging a uniform price of $2c$ would also be an equilibrium. We focus on the most profitable pure strategy equilibrium, which maximizes the range of pure strategy equilibria.

[7] Recall that we focus on the most profitable pure strategy equilibrium (there are pure strategy equilibria at lower price levels). By this we identify the necessary condition for a pure strategy equilibrium as the incentives to deviate to a price equal to v are minimal in this equilibrium.

that it serves the three closest customers (1, 2, and 3). Instead, if firm R charges the lowest price, rationing implies that it serves its three closest customers (2, 3, and 4). In both cases, firm L ends up serving its closest customer (that is, customer 1), and never its most distant (customer 4). We define the equilibrium distribution function using that each firm has to be indifferent over all prices that it plays with positive density in a mixed strategy equilibrium. The expected profit of firm L playing a price p can be expressed as

$$\pi^L(p) = \underbrace{(p-c)}_{\text{margin customer 1}} + [1 - F(p)] \underbrace{(p-2c)}_{\text{margin customer 2}} + [1 - F(p)] \underbrace{(p-3c)}_{\text{margin customer 3}}, \tag{1}$$

where $F(p)$ is the price distribution that firm R plays. We can now characterize the equilibrium distribution function F by using that firm L must be indifferent between all prices $p \in [\underline{p}, v]$, which implies that $F(p)$ is such that $\pi^L(p)$ is constant in p and equal to $\pi^L(v)$ in equilibrium. We cannot have mass points in a symmetric equilibrium if prices just below the mass point are also played with positive density, as these prices would dominate the price at the mass point. In particular, any firm that slightly undercuts a symmetric mass point gains the third most distant customer segment at essentially no cost, which is profitable at any price larger than $3c$.

There are thus no mass points at v. Consequently, a firm setting a price of v (almost surely) has the strictly highest price and thus only serves its residual demand. This yields a profit of $v - c$ from serving its closest customer. The lowest price $\underline{p}$ is defined by the price for which a firm is indifferent between the profit $v - c$ gained from charging v and charging a price that is (almost surely) the lowest, yielding a demand of the three closest customers at price $\underline{p}$. Equating the two profit levels yields $v - c = p - c + p - 2c + p - 3c$. Solving for $\underline{p}$ yields

$$\underline{p} = \frac{v + 5c}{3}. \tag{2}$$

Proposition 1. *If we restrict strategies to uniform prices, it is an equilibrium that firms mix uniform prices according to the distribution function*

$$F(p) = \frac{3p - 5c - v}{2p - 5c} \tag{3}$$

on the support $[\underline{p}, v]$. The expected equilibrium profit equals $v - c$ and there is an expected allocative inefficiency of c.

Proof. See Appendix I. □

Total surplus is $4v - 7c$, that is the maximal surplus in the case of efficient supply minus the inefficiency of c. Consumer surplus equals the difference between total surplus and the firms' profits, that is, $4v - 7c - 2 \cdot (v - c) = 2v - 5c$.

The equilibrium has the property that there is an allocative inefficiency. One customer in the middle (either customer 2 or 3) is (almost surely) supplied by the more distant and thus high-cost firm, although the low-cost firm has free capacity. The resulting inefficiency is the cost difference between the two firms for a customer in the middle. The reason for this inefficiency is that the unpredictability of prices inherent in the mixed strategy equilibrium leads to a coordination problem. One may wonder whether the restriction to uniform prices causes this outcome as uniformity implies that the prices cannot reflect the costs of serving individual customers. Indeed, we show in the next section that when costs are large in relation to the product valuation ($v/c \leq 5$), an efficient pure strategy equilibrium exists with price discrimination, but not with uniform prices. For this range, price discrimination fully solves the coordination problem. However, for higher valuation to cost ratios, allowing the firms to charge customer-specific prices does not eliminate the inefficient allocation of customers to firms. In the mixed strategy equilibrium at least part of the allocative inefficiency persists even with price discrimination.

5 Equilibria with price discrimination

In this section, we study price discrimination. We first analyze the case that each firm has either one or two units of capacity. This results in monopoly prices. We then turn to the case that each firm has four units of capacity, so that it can serve the whole market. This leads to Bertrand pricing for each customer. We finally turn to the more complex case that each firm has three units of capacity, so that it can serve more than half of the market, but not the whole market. We show that this leads to an equilibrium with mixed price strategies when the product valuation is sufficiently high in relation to the costs.

5.1 Scarce capacities of 1 or 2 units per firm

Suppose that each firm has the capacity to serve only one one or two of the four customers. As in the case of uniform pricing discussed above (Subsection 4.1), it is an equilibrium in pure strategies that each firm sets the price for each customer equal to its valuation v, and that each customer buys the good from the closest firm. Profits as well as consumer and total surplus are as stated in Subsection 4.1.

5.2 Abundant capacity of 4 units or more per firm

Suppose that each firm has the capacity to serve all four customers. Consequently, for each customer the two firms face Bertrand competition with asymmetric costs. It is thus an equilibrium in pure strategies that each firm sets the price for each customer equal to the highest marginal costs of the two firms for serving that customer, and that the customer buys the good from the firm with the lower marginal costs. This is again efficient (for given capacities) in that all customers are served by the closest firm with the lowest transport costs. Each firm makes a margin of $4c - c = 3c$ from selling to the closest customer, and $3c - 2c = c$ from selling to the second closest customer. The equilibrium profit of a firm is thus $4c$. Consumer surplus is given by $4v - 2 \cdot 4c - 2 \cdot 3c = 4v - 14c > 0$, whereas total surplus is at the maximal level of $4v - 6c$, as with uniform pricing.

5.3 Limited excess capacity of 3 units per firm

Non-existence of a pure strategy equilibrium. Suppose each firm can only serve at most three customers and both firms set prices as if there were no capacity constraints, as discussed in the previous subsection. Is this an equilibrium? For each firm, the prices charged to its two most distant customers equal its costs of supplying each of these customers ($3c$ and $4c$). Hence, a firm has no incentive to undercut these prices. Similarly, a firm has no incentive to reduce the prices for the two closest customers as these customers are already buying from the firm.

In view of the other firm's capacity constraint, the potentially profitable deviation is to set the highest possible price of v for each customer. All customers then prefer to buy from the other firm at the lower prices, which are either $3c$ or $4c$. However, as each firm only has the capacity to serve three customers, one customer ends up buying from the deviating firm at a price of v. Given the rationing rules, this is its closest customer as the price of the other firm is largest for that customer. The profit of the deviating firm is thus $v - c$. This is larger than the pure strategy candidate profit of $4c$ if $v/c > 5$.

The above condition for a profitable deviation is more restrictive by one c than the condition $v/c > 4$, which is required for contestability, and at the same time is the condition for a profitable deviation in the case of uniform prices (Subsection 4.3). With price discrimination, in the range $5 > v/c > 4$, the same equilibrium in pure strategies exist as in the case without capacity constraints.[8]

Lemma 1. *For $5 \geq v/c > 4$ and 3 units of capacity per firm, the asymmetric Bertrand, pure strategy equilibrium as without capacity constraints exists if price discrimination is feasible, whereas no pure strategy equilibrium exists with a restriction to uniform pricing.*

Mixed strategy equilibria. We now focus on the case $v/c > 5$ and solve the price game for symmetric mixed strategy Nash equilibria. Such an equilibrium is defined by a symmetric pair of joint distribution functions over the four prices of each firm. We proceed by first postulating properties and then derive results that hold for any equilibrium that has these properties. In the last step, we verify that the initially postulated properties hold in equilibrium. With this approach we are able to show that an equilibrium with the following properties exists. However, we do not exclude that mixed strategy equilibria with other properties may also exist.

Properties of the equilibria. Both firms play mixed price strategies that are symmetric across firms with prices that are weakly increasing in distance: $p_1^L \leq p_2^L \leq p_3^L \leq p_4^L$ and $p_4^R \leq p_3^R \leq p_2^R \leq p_1^R$. Every individual realization of each player's price vector in the mixed strategy equilibrium has this price order. Moreover, each individual price is mixed over the same support $[\underline{p}, v]$ and there are no mass points in the marginal distributions of the prices.

[8]For the pure strategy equilibrium we again focus on the most profitable equilibrium. Equilibria with lower prices where firms charge prices below marginal costs to customers they do not serve in equilibrium also exist.

We first provide some base results that hold for all equilibria with the above defined characteristics. We start with a property for the sales allocation, which we derive from the postulated property that firms play only weakly increasing price vectors.

Lemma 2. *If both firms play weakly increasing prices (that is $p_i^L \leq p_{i+1}^L$ and $p_i^R \geq p_{i+1}^R$, $i = 1, 2, 3$), there is zero probability that any firm will serve the most distant customer, whereas with probability 1 each firm serves its closest customer.*

Proof. See Appendix I. □

If firms play only weakly increasing prices, competition and rationing ensure that each firm always serves its home market (the closest customer). We now establish that – in such an equilibrium – price vectors where the price for the closest customer is strictly below that of the second closest customer cannot be best responses.

Lemma 3. *In any symmetric equilibrium with weakly increasing prices, the prices for the two closest customers are identical: $p_1^L = p_2^L$, and, by analogy, $p_4^R = p_3^R$.*

Proof. See Appendix I. □

The intuition for this lemma is that a firm always wins the closest customer in an equilibrium with weakly increasing prices. Each firm therefore has an incentive to increase the price for the closest customer until the property of increasing prices is just satisfied.

Lemma 4. *In any symmetric mixed strategy equilibrium with only weakly increasing prices and support $[\underline{p}, v]$ for all marginal price distributions, uniform prices are played with positive density at $\underline{p}$ and v. The lowest price is defined as*

$$\underline{p} = \frac{1}{3}v + \frac{5}{3}c < v. \tag{4}$$

Proof. See Appendix I. □

The lowest price $\underline{p}$ is defined by equating the profits of a firm at a uniform price of v ($p_i^l = v$, $\forall i$) and a uniform lowest price ($p_i^l = \underline{p}$, $\forall i$). Note that $\underline{p}$ is the same as with uniform prices. The proof of the lemma also shows that when firms play weakly increasing prices over the same range $[\underline{p}, v]$, there cannot be any mass points at v in the marginal distributions for the three closest customers.[9]

Mixed strategy equilibria with endogenously uniform prices. In the previous section we characterized the price distribution F that a firm can use to make a competitor indifferent between all uniform price vectors within the support. Let us now check whether profitable deviations are possible with non-uniform and weakly increasing price vectors. We later on verify that there are no profitable deviations with price vectors that are not weakly increasing. We have already established that changing p_4 does not change profits as long

[9]There is a degree of freedom at this stage for the fourth (most distant) customer.

as the price order is maintained (see Lemma 2), and that charging equal prices for the two closest customers is optimal (see the proof to Lemma 3). Hence, we need only to verify that there is no incentive to change p_3 individually or p_1 and p_2 together. As an intermediate result, we first show that uniform price vectors are best responses out of the set of weakly increasing price vectors in a certain parameter range.

Lemma 5. *If $v/c \geq 7$ and if price vectors are restricted to weakly increasing prices, there is a symmetric equilibrium in uniform prices with price distribution F. Instead, for a lower willingness to pay relative to the transport costs ($7 > v/c > 5$), uniform prices cannot be an equilibrium.*

Proof. See Appendix I. □

Showing in addition that if the other firm only plays uniform prices, a firm cannot profitably deviate from uniform prices with prices that are not weakly increasing establishes

Proposition 2. *If the willingness to pay is sufficiently high ($v/c \geq 7$), there exists a symmetric equilibrium in which the firms play uniform prices with distribution F in the support $[\underline{p}, v]$ (see Equation (3)). The expected profit of each firm is $v - c$ and there is an expected inefficiency of c.*

Proof. See Appendix I. □

It may not seem intuitive that firms charge uniform prices in equilibrium when costs differ and price differentiation is possible. However, a firm which faces a competitor that charges uniform prices can make sure to win the closest customer (its home market) by not charging higher prices in its home market than to other customers. When this incentive to charge high prices to close customers dominates the incentive to individually pass on the costs when costs are low in relation to the customers' valuation, we obtain an equilibrium with only uniform prices. Note that with uniform prices the margins per customer decrease in the transport costs (and are thus lower when the customer is closer to the rival). The resulting market outcome (including consumer and total surplus) is equal to that when price discrimination is not feasible (Subsection 4.3). For relatively high costs, we cannot establish equilibria with only uniform prices.

Mixed strategy equilibria with strictly increasing prices. We have established that for $v/c \leq 5$ a pure strategy equilibrium exists, whereas for $v/c \geq 7$ a mixed strategy equilibrium with uniform prices exists. Let us now investigate the intermediate range $5 < v/c < 7$ where profitable deviations from uniform prices exist. We argue again from the perspective of firm L. In the proof of Lemma 5, we show that for this parameter range each firm best responds to the uniform price distribution F with strictly increasing prices in the sense of $p_3 > p_2$ in an interval starting at $\underline{p}$. The intuition is that firm L has an incentive to increase p_3 over p_2 and p_1 when the cost differences are relatively large. We thus search for a price distribution that features strictly increasing prices.

We have established that weakly increasing price vectors are of the form $p_1 = p_2 \leq p_3 \leq p_4$ (for firm L in this case; see Lemma 3). With weakly increasing prices, a firm never serves the most distant customer, such that it is indifferent with respect to p_4 (subject to $p_4 \geq p_3$). We thus focus on $p_3 = p_4$. For marginal price deviations that maintain the order of weakly increasing prices, a firm is not capacity constrained with respect to the three closest customers. Moreover, a firm always serves its closest customer.

The (expected) profit of firm L is thus

$$\pi_L \;=\; \left(p_1^L - c\right) + \left[1 - F_2^R\left(p_2^L\right)\right]\left(p_2^L - 2c\right) + \left[1 - F_3^R(p_3^L)\right]\left(p_3^L - 3c\right),$$

where F_i^R denotes the marginal distribution from which firm R draws the price for customer $i \in \{2, 3\}$. As there is only one potential step in the price vector, we can characterize an equilibrium with increasing prices with two marginal distributions for each firm, one for the closest two customers and the other for the two most distant customers. Note that in equilibrium each firm draw the prices for the four customers jointly, such that these are weakly increasing and are consistent with the marginal distribution functions that we characterize here. Whereas the marginal distributions are defined in equilibrium, a degree of freedom remains for the joint distribution. For illustration, we construct an explicit joint distribution in Appendix II.

Let us denote the distribution of the (uniform) price for the two closest customers of each firm by $F_\mathrm{c} = F_1^L = F_2^L = F_4^R = F_3^R$, and the one for the two most distant customers by $F_\mathrm{d} = F_3^L = F_4^L = F_2^R = F_1^R$. The profit of firm L becomes

$$p_1 - c + (1 - F_\mathrm{d}(p_2))\,(p_2 - 2c) + (1 - F_\mathrm{c}(p_3))\,(p_3 - 3c), \tag{5}$$

$$\text{with } p_3 \geq p_2 = p_1. \tag{6}$$

At the top of the price support, it must still be the case that the firms play a price of v with positive density for all customers (Lemma 4). In other words, firms must play uniform prices at the upper bound of the price support. Although uniform prices are not sustainable as an equilibrium over the whole price support, uniform prices are still mutual best responses for relatively large prices. The equilibrium in uniform prices is constructed such that, given the price distribution played by firm R, firm L has an incentive to reduce p_3 individually below its other prices. This incentive is dominated by the incentive to play weakly increasing prices as a best response to uniform prices, to ensure that it serves the low-cost customer 1 in case of rationing. Together, these two incentives sustain uniform prices – but only if the cost differences are not too large in relation to the price level.[10] For low prices (that is, prices in an interval starting at the lower bound $\underline{p}$), uniform prices are not sustainable. The equilibrium price distributions need to ensure that each firm is indifferent over all weakly increasing prices in this interval as well. This is achieved when the firms plays, on average, higher prices for the two customers that are closer to the competitor. This provides an incentive for

[10]The uniform prices are constructed such that there is a marginal incentive to decrease p_3, see the proof of Lemma 5.

the competitor to play higher prices there, which balances its incentive to reduce prices for the close customers because of the lower costs.

This yields an equilibrium with piece-wisely defined distribution functions with uniform prices for high prices and price discrimination for low prices. Hence, the marginal distributions differ between the two closest and the two most distant customers for prices from $\underline{p}$ up to a certain threshold, whereas the marginal distributions are identical above that threshold up to prices of v. For any price range in which firms do not only play uniform prices, each firm must be indifferent over all prices in that range when changing the price for the two closest or the two most distant customers.

For the prices of each firm's two closest customers (p_1^L,p_2^L, p_3^R, and p_4^R), the resulting equilibrium distribution function is

$$F_{\mathrm{c}}\left(p_i^j\right) \equiv \begin{cases} \frac{3p_i^j-5c-v}{2p_i^j-5c} & \text{if } 4c < p_i^j \leq v, \\ \frac{2p_i^j-\frac{2}{3}v-\frac{10}{3}c}{p_i^j-2c} & \text{if } \ \underline{p} \leq p_i^j < 4c. \end{cases} \tag{7}$$

and for the prices of each firm's two most distant customers (p_3^L,p_4^L, p_1^R, and p_2^R), it is

$$F_{\mathrm{d}}\left(p_i^j\right) \equiv \begin{cases} \frac{3p_i^j-5c-v}{2p_i^j-5c} & \text{if } 4c < p_i^j \leq v, \\ \frac{p_i^j-\frac{1}{3}v-\frac{5}{3}c}{p_i^j-3c} & \text{if } \ \underline{p} \leq p_i^j < 4c, \end{cases} \tag{8}$$

For $v/c = 7$, the lower bound $\underline{p}$ equals $4c$ and the functions F_{c} and F_{d} coincide and equal F. This is consistent with the previously derived equilibrium for $v/c > 7$ where firms always play uniform prices on the whole support $[\underline{p}, v]$ according to the marginal (and in this case joint) distribution F (Proposition 2). Showing that it is not profitable for a firm to set prices that are not weakly increasing in response to weakly increasing prices establishes

Proposition 3. *If $5 < v/c < 7$, there exists a symmetric mixed strategy equilibrium with weakly increasing prices that satisfy $p_1 = p_2 \leq p_3 = p_4$. Each firm mixes the prices for its two closest customers according to the price distribution F_c and for the two most distant customers according to F_d, as defined in Equations* (8) *and* (7)*. All marginal price distributions are atomless with support $[\underline{p}, v]$. The expected firm profit is $v-c$ and firms play strictly increasing prices with positive probability.*

Proof. See Appendix I. □

Compared to the case of uniform prices, in the parameter range $5 < v/c < 7$ the inefficiency is smaller with price discrimination as the resulting increasing prices reflect the cost differences better than uniform prices. Consequently, the efficient allocation of each firm serving its two closest customers now occurs with a positive and thus higher probability than in the case of uniform prices. As in the case of uniform pricing, the margin per customer also decreases in the transport costs.[11] Total surplus is higher and – as profits are the same

[11] One can show that it is the case. The reason is that the price differences are small compared to cost differences. In particular, firms only play increasing prices in the range $[\underline{p}, 4c]$, which is smaller than the cost difference of c.

– consumer surplus is higher as well.

Proposition 3 does not explicitly refer to the joint price distribution of all four prices of a firm, but defines the price order for each draw and the marginal distributions. This is sufficient to characterize the equilibrium and means that firms can use any joint distribution that fulfills the conditions characterized in Proposition 3.[12]

This equilibrium exists when the cost differences are large relative to the valuation v and firms thus have an incentive to price these differences through. The pass-through of costs occurs particularly for low prices (firms play only uniform prices at the top of the price support).

Mixed strategy equilibria in non-increasing prices. In Appendix IV we show that mixed equilibria in monotonously decreasing prices (e.g., $p_1 \geq p_2 \geq p_3 \geq p_4$ for firm L) do not exist.[13] The simplification of monotonous price vectors is that then only marginal distributions are relevant for the incentives, whereas otherwise also the correlations between individual prices are relevant. For example, if both firms play non-monotonic prices, the allocation of customers to firms depends on the difference in prices relative to the difference in costs for all customers at the same time, such that any customer could be the residual customer, depending on the actual price draws. We leave these cases for future research.

Equilibria when there is a continuum of customers. Does the price structure of the equilibrium with discrete customers carry over to the case of a continuous distribution of customers?[14] As we show in Hunold et al. (2018), for moderate levels of cost differences between customers, there is an endogenous equilibrium in uniform prices also with the continuum of customers. For the same reason as with discrete customers, this equilibrium with uniform prices fails to exist for larger transport costs. In the case of discrete customers that we study in the present article, there is an equilibrium with increasing prices that has only one price step between the two customers in the middle. For the case of continuous customers, we have not characterized an equilibrium in increasing prices. However, we can provide some insights into what such an equilibrium would look like. First, similar to the case of discrete customers, there is an incentive to charge uniform prices within the home market, as these prices are capped by the incentive to play increasing prices. Secondly, there have to be multiple price steps in the price distributions, when costs differ continuously between neighboring atomistic customers. The reason is that for customers in the middle, each firm has to make the other firm indifferent over all prices in the price support for all these different customers with the different associated costs. This requires playing a different marginal price distribution for each cost level of the competitor.

[12] As an illustration, we provide a consistent pricing rule in Appendix II.

[13] However, we have not formally ruled out more complex mixed strategy equilibria with possibly non-monotonous price orders.

[14] By this we mean that there is not a discrete number of customer segments with different costs, but essentially a line with different costs at each point (costs are increasing like on a Hotelling line).

6 Pricing and efficiency with and without price discrimination

We now discuss the spatial pricing pattern, the allocative inefficiency as well as profits and consumer surplus. We focus on the interesting case of three capacity units for each firm. Depending on the relation between the transport costs and product valuation – different equilibria emerge in this case. We also highlight the effects of spatial price discrimination by comparing it to the case of a restriction to uniform prices.

Pricing patterns and conditions that lead to price discrimination. The spatial pricing pattern differs between the different equilibria that emerge for different parameter ranges. Table 1 provides an overview. It turns out that the type of equilibrium depends on the ratio of product valuation to transport costs: v/c.[15] A pure strategy equilibrium emerges if valuations are very low relative to transport costs (that is, in the range $v/c < 5$). In the pure strategy equilibrium, the spatial price pattern reflects the intensity of competition. Competition is most intense for the customers in the center (customers 2 and 3) where firms have similar cost levels. These customers pay the lowest prices.

For higher valuations ($v/c > 5$), it is profitable for each firm to deviate and set high prices, so that it only serves its home market, but earns high margins. In an intermediate range of the transport costs ($5 < v/c < 7$), there is a mixed strategy equilibrium with prices that increase in the transport costs. On average, the firms charge higher prices to the more distant half of the customers than to the nearby customers.

Restriction \ Range	$4 < v/c \leq 5$	$5 < v/c < 7$	$7 \leq v/c$
Price discrimination feasible	pure strategies, U-shaped prices efficient	mixed strategies, increasing prices less efficient	mixed strategies, uniform prices inefficient
Restriction to uniform pricing	mixed strategies uniform prices inefficient	mixed strategies uniform prices inefficient	mixed strategies uniform prices inefficient

Table 1: Equilibria by valuation relative to costs (v/c). Limited overcapacity (3 units each).

When transport costs are relatively unimportant ($7 \leq v/c$), there is a mixed strategy equilibrium in uniform prices, although price discrimination is feasible. The intuition for uniform prices is that each firm wants to charge high prices in its home market, but they also want to make sure that the price in the home market does not exceed the price in other segments in order to not be rationed in the home market (where the firm's costs are lowest). In all the cases margins decrease in costs, even when prices are increasing. Thus, we consider our result to be in line with the "standard" intuition that the prices net of transport costs (ex-works prices, margins) are lower for customers that are closer to the rival.

[15]The parameter c in our model reflects the difference in costs between the different customers and also the cost level. The results are, however, mostly driven by the difference in costs for different customers and not the level.

Efficiency. In all equilibria, all customers are served. Total surplus thus only depends on the allocation of customers to firms and the resulting transport costs (allocative efficiency).

The mixed equilibria are inefficient because firms serve distant customers with positive probability, although the closer firm would have free capacity to serve the customers at lower costs. In particular, strategic uncertainty over prices causes an inefficiency of c when firm R sets a lower price for customer 2 than firm L because R then serves the customer with its higher transport costs (and the other way around for customer 3). The probability that one of the two firms will set a strictly lower price for all customers is 100% in the case of uniform prices.

When price discrimination is feasible, firms play strictly increasing prices in the parameter range $5 < v/c < 7$. The probability of an inefficient supply is consequently lower because each firm is then more likely to charge more distant customers a higher price than the closer firm. For example, with $c = 1$ and $v = 6$ the probability is about 79%.

Lemma 6. *Price discrimination increases efficiency for intermediate levels of valuations relative to transport costs ($5 < v/c < 7$) because it induces firms to play increasing prices rather than only uniform prices. The inefficiency still occurs with positive probability when firms also mix prices with price discrimination. The probability of an inefficiency of c is strictly between $5/9$ and 1 in the case of price discrimination, and 1 in the case of uniform pricing.*

Proof. See Appendix I. □

Price discrimination also increases efficiency when costs are relatively important ($4 < v/c \leq 5$). In this range an efficient pure strategy equilibrium does not exist if firms have to charge uniform prices. However, if firms can price discriminate, an efficient pure strategy equilibrium does exist in the same parameter range. The reason is that the margins are higher in the (candidate) pure strategy equilibrium with price discrimination, such that a deviation to high prices – which eventually leads to mixed strategies – is less attractive.

Lemma 7. *For relatively low valuations ($4 < v/c \leq 5$), there exists an efficient pure strategy equilibrium when price discrimination is feasible, but only an inefficient mixed strategy equilibrium with uniform pricing.*

In summary, price discrimination increases efficiency and total surplus in the present framework for low to intermediate valuation to cost ratios, and does not affect them otherwise.

Profits and consumer surplus. In all mixed equilibria, the profit equals the profit a firm obtains when charging a price equal to the valuation v, and serving only the closest customer at a cost of c. In the pure strategy equilibrium that arises in the case of relatively high costs and price discrimination, the profit is based on the cost differences between the firms. We summarize profits and consumer surplus for the different pricing regimes and parameter ranges in Table 2.

Firms benefit from price discrimination in the range up to $v/c < 5$, whereas price discrimination does not affect the equilibrium profits for larger values of v/c.

How are consumers affected by the possibility of price discrimination? Consumer surplus (CS) equals total surplus minus the two firms' profits. Total surplus equals the gross surplus of $4v$ minus the allocative inefficiency and the efficient level of costs of $2c + 2 \cdot 2c = 6c$. In the range $5 \leq v/c \leq 7$ (middle column in Table 2), consumer surplus is higher with price discrimination than with uniform pricing because firms have the same profits in both cases, but the probability of an inefficient allocation is lower (strictly below 1) with price discrimination.

Let us now compare the cells with and without price discrimination in the first column of Table 2. Consumers also benefit from price discrimination in the range $4.5 < v/c < 5$. However, when the product valuation is very low relative to the costs ($4 < v/c < 4.5$), on average, consumers pay lower prices with uniform pricing, in spite of an inefficiency of c, as competition is more intense in the mixed strategy equilibrium that occurs in the case of uniform pricing.

In summary, our results show that for moderate to high valuation to cost ratios, consumers benefit from price discrimination, whereas firms are mostly not better off.

Restriction \ Range	$4 < v/c \leq 5$	$5 < v/c < 7$	$7 \leq v/c$
Price discrimination feasible	profit: $4c$ CS: $4 \cdot (v - 3.5c)$	$v - c$ $2v - [4 + \Pr(\text{ineff.})] \cdot c$	$v - c$ $2v - 5c$
Restriction to uniform pricing	$v - c$ $2v - 5c$	$v - c$ $2v - 5c$	$v - c$ $2v - 5c$

Table 2: Profit per firm and consumer surplus (CS) by valuation relative to costs (v/c). Limited overcapacity (3 units each).

7 Subcontracting

In the mixed strategy equilibria (Subsections 4.3 and 5.3), the transport costs are inefficiently high. With positive probability the closest (and thereby lowest-cost) firm to a customer has not won the supply contract with that customer, although it has free capacity. There is thus scope for the firm that has won the customer to subcontract the delivery to the lowest-cost firm (we also call this cross-supply). In the case of a subcontract, the firm that has initially won the supply contract charges the customer the agreed price, and it compensates the lowest-cost firm for actually supplying the product. The resulting efficiency rent can be shared between the two firms. It thus seems that firms should always conclude such a subcontract in such situations.

In the established literature subcontracting always takes place whenever a cost saving is feasible if firms decide on subcontracting after they have set prices or quantities (see Section 2). We add to this literature the insight that even when cost-reducing cross-supplies are feasible after the firms have set their prices, they may nevertheless prefer not to engage in cross-supplies. The reason is that a cross-supply relaxes the capacity constraints of the firm that has set aggressive prices and thereby leads to lower equilibrium price realizations – even after prices have been set. The firms can thus be better off not engaging in such cross-supplies. We formally characterize this insight in the following extended game:

(i) firms set customer prices – as before,

(ii) firms see each others' prices and can agree to cross-supply customers,

(iii) customers are allocated to the firms – according to prices and capacity constraints.

In stage (ii) a firm anticipates that it will serve the customers for which it has the lowest price and free capacity. There is scope for a cross-supply if a firm has set the lowest price for a more distant customer that could be served at lower costs by the closer firm. If the firms agree on a cross-supply, the cross-supplier serves the customer from its location as a subcontractor, incurs the associated transport costs, and receives a transfer from the low-price firm. The customer still pays the original price to the low-price firm.

Cross-supplies yield an efficiency rent for the firms whenever it reduces their total costs. However, there is the potential disadvantage for the cross-supplier because the competitor receiving a cross-supply has one more unit of free capacity, which it can use to supply another customer. For example, consider that firm R sets a uniform price of v for all customers, and firm L a strictly lower uniform price $p_L < v$. Without subcontracting, L will simply supply customers 1, 2, and 3 up to its capacity limit, and firm R will serve the residual customer 4. Consider that firm R agrees to supply customer 3 by means of a subcontract with L. Now firm L has one more unit of free capacity. Consequently, customer 4 will be allocated to firm L in stage (iii) as L charges a strictly lower price and – due to the subcontract for customer 3 – still has an unused unit of capacity. This implies that firm R would forgo its residual demand profit of $v - c$. Indeed, the firms could agree to also subcontract for customer 4 to save the cost difference $4c - c$ as it is inefficient that firm L supplies the most distant customer 4. This cost saving is, however, only a side-effect of the cross-supply for customer 3, as otherwise firm L would be capacity constrained and firm R would serve customer 4 anyway – at its low transport costs of c. The only effective cost saving is thus that of $3c - 2c = c$ for customer 3. This needs to be traded off against the lost revenues when firm L serves customer 4 at a price of $p_L < v$ instead of firm R serving that customer at the higher price $p_R = v$. Taken together, a cross-supply can only yield a Pareto-improvement for the firms when the cost saving on customer 3 is higher than the lost revenue on customer 4: $c > p_R - p_L$.

Subcontracting reduces total costs by the cost difference between the two firms for each subcontracted customer. Depending on the expected payments between the firms, subcontracting changes the perceived cost when competing in stage (1). We follow Kamien et al. (1989) in assuming that the firms make take-it-or-leave-it offers. The firm that makes the offer has all the bargaining power as it can choose the terms of the contract such that the other firm does not get any additional rent. It can thus extract all the additional profits that subcontracting yields. We proceed by analyzing the two cases of either

(a) the firm that has set the lower price and demands a cross-supply, or

(b) the potential cross-supplier, i.e., the firm has set the higher price, but has lower costs for that customer,

making the offer and characterize the resulting equilibria.

For the analysis of subcontracting we abstract from price differentiation and only consider uniform price vectors. For uniform price vectors, there are no pure strategy equilibria if $v > 4c$ (the contestability assumption).[16] We have already established that without subcontracting uniform prices endogenously emerge for a large parameter range (Proposition 2). With subcontracting, however, depending on timing and the bargaining power, different non-uniform price patterns might emerge and there appear to be multiple equilibria in each case. Abstracting from price differentiation simplifies the analysis and comparison by yielding a unique equilibrium in each case. We conjecture that the results obtained for uniform prices qualitatively also hold for non-uniform prices.

(a) The firm demanding the cross-supply gets the additional rents

Consider that the firm with the lower (uniform) price offers the subcontract. It can make an offer that extracts the additional rents from subcontracting. Consequently, the low-price firm's perceived costs are lower because it can serve two most distant customers at low cost by means of a subcontract. This is particularly relevant for the third closest customer, as without subcontracting the most distant customer is not served by the low-price firm, due to its capacity constraint. Note that with uniform prices, if a firm supplies one customer as a cross-supplier of the low-price competitor, it will not sell to any customer directly – not even to the closest. The reason is that the cross-supply frees the capacity of the competitor, which enables it to serve the remaining customer for which it has also set the lowest uniform price. Hence, any equilibrium subcontract foresees two cross-supplies, such that the firm with the lower price does not supply the two most distant customers itself.

With uniform prices played by the other firm according to the cdf $F(p)$, the expected profit of a firm is given by

$$\begin{aligned}\pi(p) = {} & p - c + (1 - F(p))\,(p - 2c) + (1 - F(p))\,(p - 3c) \\ & + (F(p+c) - F(p))\left(c - \int_p^{p+c} x \frac{f(x)}{(F(p+c) - F(p))}\,dx + p\right).\end{aligned}$$

The last term is new here compared to the case without cross-supplies (see Equation (1)). It states that with a probability of $(F(p+c) - F(p))$ the other firm sets a price in-between p and $p+c$. In this case the cost saving of c on the third closest customer is larger than the foregone revenue of the cross-supplier on the most distant customer. The expected lost revenue for this case is the difference of the average price in the range p to $p+c$ and p.

[16] This holds without and with subcontracting. With subcontracting, the incentives to deviate from the candidate equilibrium in pure strategies are even stronger as either the candidate equilibrium profits are lower (case (a)) or the residual profits are larger (case (b)).

Proposition 4. *If subcontracting takes place before rationing, and if the firm demanding the cross-supply gets the additional profits, and if only uniform prices can be played, there is a symmetric mixed strategy equilibrium with an atomless price distribution that has an upper bound of v. The expected profit in this symmetric equilibrium is $v - c$, customers benefit from subcontracting, total surplus increases, but the firms choose not to realize all feasible efficiency-enhancing cross-supplies.*

Proof. See Appendix I. □

(b) The cross-supplier gets the additional rents

Consider that the firm with the higher uniform price offers the subcontract. It can make an offer which extracts all the additional rents that arise from subcontracting. Consequently, setting a high price becomes more attractive because charging a higher price than the competitor yields not only a residual demand profit, but also an additional income from subcontracting arises with positive probability. The expected equilibrium profit is thus larger when the cross-supplier determines the terms of the contract.

With uniform prices played by the other firm according to the cdf $F(p)$, the expected profit of a firm is now given by

$$\pi(p) = p - c + (1 - F(p))(p - 2c) + (1 - F(p))(p - 3c) \\ + (F(p) - F(p-c)) \left(c + \int_{p-c}^{p} x \frac{f(x)}{(F(p) - F(p-c))}\, dx - p \right).$$

The last term is different compared to the case when the firm that has set the lower price gets the rents from subcontracting. It states that with probability $(F(p) - F(p - c))$ the other firm will set a price in-between $p - c$ and p. In this case the cost saving of c on the second closest customer is larger than the forgone revenue on the closest customer (which the firm with the higher price otherwise serves as residual demand). The expected lost revenue for this case is the difference of p and the average price in the range $p - c$ to p.

The expected profit of a firm choosing a price of v is

$$\pi(v) = v - c + (1 - F(v - c)) \left(c + \int_{v-c}^{v} x \frac{f(x)}{(F(v) - F(v-c))}\, dx - v \right), \tag{9}$$

which defines the expected equilibrium profit. Note that the second term is the efficiency gain minus the positive externality on the customers, due to the price reduction for the closest customer. The size of that term increases in the density of prices close to v, i.e., in the interval $[v - c, v]$.

Proposition 5. *If subcontracting takes place before rationing, and if the cross-supplier gets the additional rents, and if only uniform prices can be played, there is a symmetric mixed strategy equilibrium with an atomless price distribution that includes v in its support. The expected profit of a firm is strictly larger than the profit $v - c$ without subcontracting, but not* *larger than v. Total surplus is higher than without subcontracting. If $v > {}^9\!/_2 c$, the firms do not realize all efficiency-increasing subcontracts.*

Proof. See Appendix I. □

Consumer surplus equals total surplus minus total profits. The profit of each firm increases by the expected payments from subcontracting at the price of v, which could be up to c for each firm (see Equation (9)), whereas total surplus increases by at most c, which is the efficiency gain if subcontracting always takes place. Consumer surplus could thus decrease by up to c.

Irrespective of whether the cross-supplier or the receiver of the cross-supply gets the additional rents of the cross-supply, production becomes more efficient. Whether customers benefit through lower prices depends on the bargaining power among the subcontracting parties. If the firm that offers the lower price is rewarded by being able to extract the cross-supplying rents, customer prices are lower than without subcontracting. If the firm with the higher price can extract the additional rents, prices potentially increase.

We have analyzed the case that firms subcontract after prices are set, but before rationing takes place. This is plausible as the cross-supply changes the available capacities of the firms. An interesting result is that firms do not cross-supply each other in certain cases although this would reduce costs.[17] The reason is that a firm which supplies a capacity constrained competitor has to fear that the competitor can serve additional customers once it receives a cross-supply, as this frees up capacity. For a potential cross-supplier it can therefore be optimal to reject a cross-supply request. Theoretically, this problem can be solved if the firm receiving the cross-supply agrees to not use the additional capacity for serving the cross-supplier's customers. However, such an agreement would by its nature restrict competition and is therefore potentially illegal (cartel prohibition).[18] One may wonder whether such an agreement should be legal from a welfare point of view. A major concern regarding such an exemption from the cartel prohibition is potentially that it could be used to restrict competition far beyond the particular context of an efficiency increasing cross-supply.

8 Endogenous capacities

In this section we investigate which capacity levels firms optimally choose before competing in prices. To focus on the strategic capacity trade-off, we assume that capacity is costless and that a firm chooses the lower level if two different levels yield the same profit. We first show that when demand is certain with a level of four, it is an equilibrium that each firm chooses a capacity of two. This result of no excess capacities when demand is certain resembles

[17] If the allocation of customers to firms is instead finalized before firms can subcontract, all cost-reducing cross-supplies will take place. This would correspond to a more static market, where capacities that are freed in the process of subcontracting cannot be brought back to the market. A proof of this case is available and can be provided.

[18] For the European Union, Article 101 (1) of the Treaty on the Functioning on the European Union prohibits anticompetitive agreements.

Kreps and Scheinkman (1983). At the same time, it is plausible that demand fluctuates to some degree in certain – if not most – markets. We show that when demand is uncertain, overcapacity can result in equilibrium. Moreover, it is noteworthy that excess capacity can also arise as the result of collusion (see, for instance, Fershtman and Gandal, 1994). After a breakdown of collusion, competition with overcapacity might therefore occur even absent demand uncertainty.

For the analysis of endogenous capacity under competition, we initially assume that firms play uniform prices. We extend the analysis to the case of price discrimination in Annex V and get essentially the same insights.[19]

Certain demand and no excess capacity. Consider first that the level of demand of four is certain and common knowledge when firms choose capacities. As assumed so far, there are four customers, each with unit demand. Each firm can choose an integer capacity of $1, 2, 3$ or 4 units. It is never profitable for a firm to choose a capacity level that exceeds the total demand of four units as it can obtain the same profit with a capacity level of four – the maximal level of demand it can possibly serve. To determine a lower bound, note that it is not an equilibrium that total capacity is below total demand. In such a situation each firm has an incentive to increase capacity (at least) until total capacity equals total demand, as firms remain local monopolies up to this level. Hence, the total equilibrium capacity is at least four, but not above eight.[20]

We now show that total capacity equals total demand in equilibrium when there is no demand uncertainty. In particular, it is an equilibrium that each firm has two units of capacity.[21] For this argument, we have to derive the profit levels for deviating capacity choices.

We have already established the equilibrium and expected profits for the symmetric capacity configurations of one to four units per firm in Section 4). Let us now consider that total capacity is larger than four and thus exceeds demand. Consider that one firm, say L, has the weakly larger capacity, denoted by l. Denoting the capacity of firm R with r, the cases that are left to inspect are $(l = 3, r = 2)$ and $(l = 4, r = 2)$.[22]

Let us start with the capacity configuration $(l = 3, r = 2)$. When its competitor plays uniform prices, each firm can guarantee itself a minimum profit of serving its closest customer(s) at a price of v. For firm L, this minimum profit is $v - c + v - 2c = 2v - 3c$, and for firm R it is $v - c$. These profits define the lowest prices each firm is willing to play as uniform prices. For firm L the lowest price is defined by the indifference in terms of profits between the highest and the lowest price: $2v - 3c = 3\underline{p}_L - 6c$. This yields $\underline{p}_L = \frac{2}{3}v + c$.

[19] We can establish that also with price discrimination neither increasing nor reducing the capacity by a unit is a profitable deviation from the Nash equilibrium choice of capacities when prices are restricted to uniform prices.

[20] For this argument we implicitly focus on pure strategy equilibria in the capacity game.

[21] Given the different transport costs, this is the Pareto-dominating outcome for firms and also maximizes total surplus.

[22] We have calculated the equilibrium profits for all possible capacity configurations, even those that we do not present here, and present all results in Table 3.

Similarly, the lowest price of firm R is derived from $v-c=2\underline{p}_R-3c$, which yields $\underline{p}_R=\frac{v}{2}+c$. The prices derived for the two firms are not identical, and the lowest price that is played by both firms is defined by $\max(\underline{p}_L,\underline{p}_R)=\underline{p}_L\equiv\underline{p}$. Thus, at $\underline{p}$ firm R makes a profit of $\pi_R(\underline{p})$, which exceeds the profit at a price of v if L does not play v with positive probability (that means a mass point at v). Hence, firm L playing a mass point at v is necessary and yields equilibrium profits of $\pi_L(v)=2v-3c$ and $\pi_R(\underline{p}_L=\underline{p})=\pi_R(v)=\frac{4}{3}v-c$. Comparing these profits with the profits at capacity levels of $(l=3,r=3)$ shows that each firm has a weak incentive to reduce the capacity from three to two. The incentive is strict if capacity is costly. Analogously, with capacities $(l=2,r=2)$ a firm has no incentive to increase the capacity from two to three as the profit does not increase. See Table 3 for a summary of the profits.

For completeness, let us check whether more drastic deviations from capacities of $(l=2,r=2)$ are profitable by considering the case $(l=4,r=2)$. In this case, firm R cannot unilaterally ensure for itself a positive profit. Instead, as before, firm L can guarantee itself a profit of $2v-3c$. Hence, also the lowest price L is willing to play is $\underline{p}=\frac{2}{3}v+c$ as before. This is the maximum of the lowest prices, as R would be willing to play lower prices because it has no "guaranteed" positive profit. This again yields expected profits of $\pi_L(v)=2v-3c$. As before, the profit of R is defined by the profit it obtains at $\underline{p}$, the lowest price L is willing to set. Hence, $\pi_R(\underline{p})=\frac{4}{3}v-c$ must be the expected profit R obtains at any price it plays, also at v. This again requires a mass point at v for L, as otherwise playing v would yield zero profit for R and would thus not occur.

Summary. If one considers a situation of two capacity units for each firm, no firm has an incentive to lower its capacity. This would simply leave valuable demand unserved. Also, no firm has an incentive to increase capacity. If a firm has more than two units of capacity, while the other firm has a capacity of two, it still makes the same profit – gross of capacity costs, as with two units of capacity.

In Appendix V we solve the price equilibria of the game allowing for price discrimination. Interestingly, the resulting profits for the capacity levels $(2,2)$ — and all capacity deviations by one unit from this – result in the same profits obtained without price discrimination. However, there are sometimes differences for even higher aggregate capacity levels, depending on the parameters.

		Firm R			
	Capacities	1	2	3	4
Firm L	1	$v-c,\ v-c$	$v-c,\ 2v-3c$	$v-c,\ 3v-6c$	$\frac{3}{4}v,\ 3v-6c$
	2	$2v-3c,\ v-c$	$2v-3c,\ 2v-3c$	$\frac{4}{3}v-c,\ 2v-3c$	$\frac{4}{3}v-c,\ 2v-3c$
	3	$3v-6c,\ v-c$	$2v-3c,\ \frac{4}{3}v-c$	$v-c,\ v-c$	$\frac{3}{4}(v-c),\ v-c$
	4	$3v-6c,\ \frac{3}{4}v$	$2v-3c,\ \frac{4}{3}v-c$	$v-c,\ \frac{3}{4}(v-c)$	$3c,\ 3c$

Table 3: Profits for different capacity levels when total demand is 4.

Uncertain demand and excess capacity. To understand how capacity levels of three units per firm can arise when total demand is four, consider that there are two possible states of demand: In one state total demand is four units, in the other state total demand is six units because the four customers each demand 1.5 units. Assume that if the demand state is six units and each firm can only supply less than three units, but more than 1.5 units, it will partially serve the second customer up to its capacity limits. Both states occur with positive probability. As argued before, if the state of a demand of four units would occur with certainty, the capacity choice in equilibrium would be two units per firm. Analogously, if the state of a demand of six units would occur with certainty, the capacity choice in equilibrium would be three units per firm. If there is uncertainty, the capacity levels $\{l = 3, r = 3\}$ are an equilibrium as long as the high demand state is sufficiently likely. Note that having a capacity of three instead of two does not reduce the profits (gross of capacity costs) – if the other firm has a capacity of three. Nevertheless, a trade-off occurs as each firm can increase its profit in case of a demand of four by reducing the capacity from three to two, if the other firm has a capacity of three units. The profit increases by $\left(\frac{4}{3}v - c\right) - (v - c) = \frac{v}{3}$. This provides an incentive to choose a capacity of two instead of three. In case of high demand, reducing the capacity from three to two, when the other firm has a capacity of 3, reduces the sales to customers to which the firm could charge a monopoly price of v as there are no excess capacities. The profit loss from reducing the capacity by one is $v - 2c$. If the probability for the low demand state is α and that for the high demand state is $1-\alpha$, there is an equilibrium in which both firms choose a capacity of three if $\alpha\frac{v}{3} < (1-\alpha)(v - 2c) \Leftrightarrow \alpha < \frac{3v-6c}{4v-6c}$. For instance, for $v = 7c$, this yields $\alpha < \frac{15}{22}$.

If we allow for price discrimination, the profits for most cases do not change. In particular, the profits for any combination of $(2,2)$, $(3,2)$, and (3,3) are identical. This yields the same trade-offs when choosing capacities under demand uncertainty. A difference is that with price discrimination the profits when demand is four, in the case $(4,3)$ and $(4,4)$ are larger. However, these profits are still never larger than those in the case $(3,3)$, for both, four and six units of demand.

Summary. When firms choose their capacities in view of uncertain demand, firms trade-off additional sales in high demand states with lower prices in lower demand states, due to overcapacities. The resulting capacity choices tend to yield excess capacity in the low demand states, and no excess capacity in the high demand states.

9 Conclusion

Strategic uncertainty and inefficient competition. We have characterized a competitive equilibrium with prices that weakly increase in the costs of serving the different customers. This, together with limited overcapacities, yields the outcome that a firm typically serves its closest customers (its "home market"). However, intermediate customers are sometimes served by a more distant firm with higher costs, although there is a firm with lower costs and free capacities. Importantly, this occurs when location and customer-specific pricing is feasible and firms price discriminate in equilibrium. The reason is that price competition in the presence of capacity constraints results in unstable prices. As one competitor does not know which prices the other competitor will ultimately charge, there is strategic uncertainty.

The result of an allocative inefficiency due to strategic uncertainty arises although we have tilted the model toward efficient supplies. In particular, we allow firms to price discriminate according to location, assume efficient rationing, and focus on equilibria with weakly increasing prices, such that the costs tend to be reflected in the price schedule. We conjecture that alternative rationing rules can yield even more inefficiencies as prices may be even less aligned with costs.

Cross-supplies. Cross-supplies between suppliers can reduce the allocative inefficiencies of price competition with strategic uncertainty. However, from a welfare perspective, cross-supplies are a double-edged sword. On the one hand, they can clearly increase efficiency by reducing costs. On the other hand, they can dampen competition as a supplier that anticipates becoming a cross-supplier has fewer incentives to aggressively compete for the customers in the first place. We find that cross-supplies do not harm customers through higher prices if the cross-supplier makes no profit on its subcontract (or a sufficiently low one). Consequently, subcontracts where the cross-supplier sells to a competitor at marginal costs, tend to be pro-competitive, whereas arrangements which foresee that the cross-supplier earns a significant profit on a cross-supply have the potential to dampen competition. What matters here is the profit obtained from an additional cross-supply, and not necessarily the overall profitability. A subcontract arrangement can therefore be pro-competitive if the cross-supplier is remunerated upon signing a framework agreement with a fixed fee and in turn conducts cross-supplies at marginal costs.

Efficiency increasing cross-supplies may not always take place because firms fear additional competition. In particular, we have shown that when an almost capacity constrained firm asks the unconstrained firm for a cross-supply, the unconstrained firm may deny this supply. The reason is that the cross-supply would endow the demanding firm with additional capacity, which can intensify competition for other customers.

Bertrand-Edgeworth arguments in competition policy. Various competition policy cases feature homogeneous products with significant transport costs for which location or customer-based price discrimination is common. Several decisions make explicit references to Bertrand-Edgeworth models, but without taking geographic differentiation and customer-specific pricing into account. For instance, in relation to the merger M.6471 OUTOKUMPU/INOXUM in 2012 the European Commission (Commission) noted that "*one of the main criticisms of the Notifying Party of this [Bertrand-Edgeworth] model of aggressive competition is that it tends to predict more competitive prices pre-merger than the observed pre-merger price*". The Commission acknowledged that there may be reasons for less intense competition that have not been accounted for: "*Customers may have other preferences for a specific firm (e.g. geographic proximity, preferences for a specific firms products based on* *quality concerns or experiences; or on-going business relationships or contracts etc).*"[23] Our model allows for both customer-firm-specific (transport) costs as well as prices and can thus aid the analysis in future cases.

Our model can also help to assess whether firms compete with each other or coordinate their sales activities. In the assessment of the merger M.7009 HOLCIM/CEMEX WEST in 2014 the Commission argued "*that the most likely focal point for coordination in the cement markets under investigation would be customer allocation whereby competitors refrain from approaching rivals' customers with low prices. Under such a coordination scenario, the sizable transport costs for cement would lead to a general allocation of customers based on proximity to a given plant. The Commission has thus investigated the hypothesis that cement competitors might face limited incentives to enter significantly into competitors' geographic strongholds...*"[24] The Commission concludes that "*given the low level of differentiation across firms and the existing overcapacities, it is difficult to explain the observed level of gross margins as being the result of competitive interaction between cement firms.*"[25] As a supporting argument, the Commission refers to a Bertrand-Edgeworth model with constant marginal costs and uniform pricing.[26]

Our model makes several predictions which can be related to the above reasoning: Even with overcapacities of 50%, we find that in a competitive equilibrium firms may always exclusively serve their home markets, and that at prices above the costs of the closest competitor. Firms set high prices in the home markets of rival firms, although a unilateral undercutting there seems rational in view of their overcapacities. Such a pattern is difficult to reconcile with previous models of competition. On the one hand, without capacity constraints, the logic of asymmetric Bertrand competition predicts prices equal to the marginal costs of the second most efficient firm. On the other hand, the typical Bertrand-Edgeworth model with capacity constraints, but uniform costs and uniform pricing, does not explain spatial price differentiation and customer allocation.

Both the Bertrand model with customer-specific costs and prices, but without capacity constraints, as well as the homogeneous Bertrand-Edgeworth model tend to predict significantly lower competitive margins than our model, which can incorporate all of these market features simultaneously.[27] When using an over-simplified model, a competition authority may thus wrongly conclude that the observed market outcome cannot be the result of competition, but rather of coordination among the suppliers. To answer the question of whether suppliers are indeed coordinating or competing, the new model – which allows for geographic differentiation, location-specific pricing, and capacity constraints at the same time – could therefore improve the reliability of competition policy assessments. See Appendix III for a numerical presentation of this argument.

There is plenty of scope for further research. On the more theoretical frontier, avenues

[23] See paragraphs (725) and (407) in the Commission's decision in M.6471 OUTOKUMPU/INOXUM, and more generally Annex IV of the decision for the Bertrand-Edgeworth modeling.

[24] See paragraphs (167) and (168).

[25] Paragraph (178).

[26] See the European Commission decision M.7009 HOLCIM/CEMEX WEST, fn. 195.

[27] See Appendix III for sample calculations of unexplained excess margins when using the different models.

for future research include studying equilibria with non-uniform prices in a setting with continuous demand as well as alternative rationing rules. On the more applied frontier, reformulating the model to allow for more than two firms and simulating the effects of mergers in such a setting appears to be of particular interest.

Appendix I: Proofs of lemmas and propositions

Proof of Proposition 1. Using that the expected profit for any price vector that is played in equilibrium must be $v - c$, we can solve for $F(p)$ in the symmetric equilibrium in uniform prices by equating the profit from (1) with $v - c$:

$$(p - c) + [1 - F(p)](p - 2c) + [1 - F(p)](p - 3c) = v - c$$

$$\Leftrightarrow F(p) = \frac{3p - 5c - v}{2p - 5c}.$$

As the prices of the two firms are almost surely not identical, there is an inefficiency of c as one of the two intermediate customers (2 or 3) is served by a firm with transport costs that are higher by c than those of the more efficient firm, which in these cases has set higher prices and still has unused capacity. □

Proof of Lemma 2. There are two cases: either the capacity constraint of a firm is binding (and there is rationing), or it is not binding:

If each firm has the lowest price for at least its closest customer, the customer closest to each firm is won by that firm as there is no rationing.

Instead, if all the prices of one firm are pairwise below the prices of the other firm ($p_i^L \leq p_i^R$ or $p_i^R \geq p_i^L, \forall i \in \{1, 2, 3, 4\}$), the firm with the lowest prices serves its closest three customers up to the capacity limit; the most distant customer is served by the firm with the high prices. This minimizes the prices that customers pay and thus maximizes their surplus, in line with the rationing rule. □

Proof of Lemma 3. There are two cases to distinguish:

First, suppose that $p_2^L \leq p_2^R$. Given a weakly increasing price order of firm R, firm L wins the first customer with a price of $p_1^L = p_2^L$ as $p_1^R \geq p_2^R \geq p_2^L$. Consequently, $p_1^L \leq p_2^L \leq p_1^R$. Hence, there is no incentive to charge a price $p_1^L < p_2^L$, as $p_1^L = p_2^L$ ensures a higher margin without losing demand.

Second, suppose that $p_2^L > p_2^R$. Given weakly increasing price orders, this means that R also has the lowest prices for customers 3 and 4. In this case firm L will serve customer 1 even if it has a higher price than R, as R – given the rationing rule – serves its three closest customers, such that customer 1 only has the option to buy from L or not at all. In this case setting $p_1^L < p_2^L$ is strictly worse than $p_1^L = p_2^L$.

In both cases the price relation $p_1^L < p_2^L$ – and by analogy $p_4^R < p_3^R$ – is strictly worse and thus dominated by equal prices for the two closest customers, which establishes the lemma. □

Proof of Lemma 4. Recall that when suppressing the superscript we mean prices of firm L. The postulated property of equal price supports together with weakly increasing prices imply that if the price for the closest customer of firm L is at the maximal level, $p_1 = v$, the other three prices of firm L must also equal v, such that it plays $p_1 = p_2 = p_3 = p_4 = v$ with positive joint density. Similarly, firms play the lower bound $\underline{p}$ with positive density, which implies that firm L plays $p_4 = \underline{p}$ with positive density. Again, if firm L sets $p_4 = \underline{p}$, weakly increasing prices imply that it plays $p_1 = p_2 = p_3 = p_4 = \underline{p}$ with positive joint density. Thus firms play uniform prices with positive density, at least at the boundaries of the price support. We can now establish that there are no mass points, given that firms play weakly increasing symmetric price vectors over the same support for each customer, and determine the lowest price $\underline{p}$ explicitly. For all customers the upper bound price v is played with positive joint density by both firms. In turn, we can exclude that there are mass points at v in the price distribution of the closest three segments. By symmetry, only symmetric mass points have to be considered. To rule these out, note that each firm would want to deviate from a symmetric mass point by moving some joint density to a price that is just below v, thereby increasing demand by a discrete amount at virtually no cost. As there is no mass point for v, a firm can realize a profit of $\pi(v) = v - c$ with probability one by choosing a price of v for all customers. As all price vectors played in equilibrium must yield identical expected profits, the expected profit must equal $v - c$ for each price vector that is played with positive density. This must also hold for the uniform price vector with the lowest price $\underline{p}$, which is played with a positive joint density – as argued before. This yields the profit identity $\pi\left(\underline{p}\right) = \pi(v)$, which implies $\left(\underline{p} - c\right) + \left(\underline{p} - 2c\right) + \left(\underline{p} - 3c\right) = v - c$ and defines the lowest price $\underline{p} = \frac{1}{3}v + \frac{5}{3}c < v$. □

Proof of Lemma 5. As established in Lemma (3), a best response to weakly increasing prices with weakly increasing prices has the property $p_1 = p_2 \leq p_3 \leq p_4$. As the most distant customer is never served, we restrict our search for best responses to price vectors with p_4 equal to p_3 (as $p_4 = p_3$ is always a best response). This leaves only one critical price step in the best responses: the potential step between prices p_2 and p_3. We first verify that there is no incentive to deviate from uniform prices by increasing the price for customer 3 individually, while maintaining the order of weakly increasing prices. We afterwards verify that only weakly increasing prices are best responses to uniform prices. Note that changing the price of customer 3 (and 4) in a way that the order of weakly increasing prices is maintained does not affect the expected profits of the firm with customers 1 and 2.

There is an underlying incentive for a firm to charge higher prices to more distant customers as these are more costly to serve. To see this, note that the expected profit for firm L from serving one customer $i \in \{1, 2, 3, 4\}$ with the lowest price (i.e., without residual demand profits and in the absence of capacity constraints) is given by $[1 - F(p_i)](p_i - i \cdot c)$. Differentiating with respect to p_i yields

$$[1 - F(p_i)] - f(p_i)\, p_i + f(p_i) \cdot i \cdot c. \tag{10}$$

The marginal profit for firm L increases in the distance i. There is thus a natural incentive to set higher prices for more distant customers. Hence, if there is no incentive to increase p_3, then, for the same price distribution played by the other firm, there is also no incentive to increase p_2 (and p_1). We evaluate the marginal profit in (10) for customer 3 by substituting

$i = 3$, and for f and F from (3). From this we derive the parameters for which the marginal profit is negative, such that a marginal price increase of only p_3 is not profitable:

$$\left[1 - \frac{3p - 5c - v}{2p - 5c}\right] - \frac{2v - 5c}{(2p - 5c)^2}(p - 3c) < 0$$

$$\Longrightarrow \; (2p - 5c)(v - p) - (2v - 5c)(p - 3c) \;<\; 0. \tag{11}$$

For $p = \underline{p}$ the marginal profit condition reduces to $-\frac{1}{3}v + \frac{7}{3}c < 0$. This is equivalent to $v > 7c$. Moreover, we show that the second derivative of the profit for customer 3 is negative in the relevant range. The second derivative is given by

$$2(v - p) - (2p - 5c) - (2v - 5c) \; = 2v - 2p - 2p + 5c - 2v + 5c = -4p + 10c.$$

This second derivative is already negative at the lower bound price of $\underline{p}$ for the lowest possible value for v of $5c$, above which there are mixed strategy equilibria. It is also negative for all larger prices up to v. The profit function is thus strictly concave in the relevant range. This implies that whenever the marginal profit (10) is negative at $\underline{p}$, it is negative for the prices above $\underline{p}$. □

Proof of Proposition 2. Consider that firm R plays uniform prices. Suppose that firm L chooses prices that are not weakly increasing. In that case there is a pair p_j, p_k of prices of firm L with $p_k < p_j$ and $j < k$. In all such cases it is at least as profitable to switch p_k and p_j such that $p_k > p_j$. To see this, consider the three possible outcomes: the uniform price of R, p^R, is above, below, or in-between the price pair. Firstly, if $p^R > p_j > p_k$, switching p_j and p_k weakly increases profits. In particular, it is profit-neutral if the capacity constraint is not binding, and strictly profit-increasing if customer j is rationed, as this reduces the costs for the customers served without affecting the average price level of the customers that are served. Secondly, if $p_R < p_k < p_j$, switching p_k and p_j can affect which customer is served by firm L as its residual demand (if p_k is the lowest price) and is thus weakly profitable because serving customer j has lower costs. Thirdly, if $p_j > p_R > p_k$, the capacity constraint is not binding for either firm. In this case switching the prices to a weakly increasing price order is always profitable as it changes the customer that is served by firm L from k to j, with less costs and without changing the prices that are realized. □

Proof of Proposition 3. The marginal distribution functions F_{c} and F_{d} in the mixed strategy equilibrium are defined by an indifference condition. A marginal change in the prices $p_1 = p_2 \equiv p_{12}$ must not change the profit in Equation (5):

$$1 + 1 - \underline{F}_{\mathrm{d}}(p_{12}) - \underline{f}_{\mathrm{d}}(p_{12})(p_{12} - 2c) = 0. \tag{12}$$

This condition defines the marginal distribution of the two distant prices in the range where firms do not always play as part of uniform price vectors in equilibrium. We denote this part of the distribution function by $\underline{F}_{\mathrm{d}}$. To obtain $\underline{F}_{\mathrm{c}}$, we differentiate the profit in (5) with respect to p_3 and obtain the marginal indifference condition

$$1 - \underline{F}_{\mathrm{c}}(p_3) - \underline{f}_{\mathrm{c}}(p_3)(p_3 - 3c) = 0. \tag{13}$$

Let us first solve the differential equations (12) and (13). The solution to the differential equation (12) is

$$\underline{F}_c(p) = \frac{2p - k_c}{p - 2c}.$$

At the lower bound price $\underline{p}$, it must be that the distribution function has a value of 0. This implies $k_c = 2\underline{p} = \frac{2}{3}v + \frac{10}{3}c$ and thus

$$\underline{F}_c(p) = \frac{2p - \frac{2}{3}v - \frac{10}{3}c}{p - 2c}.$$

Non-uniform prices cannot be played globally as $\underline{F}_c(v) > 1$ for $v \in [5c, 7c]$. The solution to the differential equation (13) is

$$\underline{F}_d(p) = \frac{p - k_d}{p - 3c}.$$

This distribution function must also be 0 at $p = \underline{p}$, which implies $k_d = \frac{1}{3}v + \frac{5}{3}c$ and yields

$$\underline{F}_d(p) = \frac{3p - v - 5c}{3p - 9c}.$$

Note that $\underline{F}_d(v) < 1$ for $v \in [5c, 7c]$, such that $\underline{F}_d$ can only describe a part of the price distribution. However, note that $\underline{F}_d(4c) = F_d(4c) = F(4c)$, with $4c > p^I$, where p^I is such that for prices exceeding p^I, uniform prices are feasible even in the parameter range $7c > v > 5c$. This implies that marginal distribution functions F_d and F_c that support the equilibrium exist. Firms play increasing prices from $\underline{p}$ to $4c$ and uniform prices from $4c$ to v.

It is left to verify that there is no profitable drastic deviation that overturns weakly increasing prices. Consider firm L for the argument. Given firm R plays weakly increasing prices with $p_1^R = p_2^R$ and $p_3^R = p_4^R$ according to the equilibrium distributions F_d and F_c. We show next that every best response to that strategy has weakly increasing prices.

Suppose to the contrary that L violates weakly increasing prices by playing $p_j > p_k$ with $j < k$. First, let us investigate the case $p_4 < p_3$. By the same logic as for uniform prices (proof of Proposition 2), it is profitable to switch the two prices, such that the high cost customer faces the higher price. As R sets identical prices for its two closest customers, only the same three outcomes of the uniform pricing case can occur. Moreover, the same logic holds for p_1 and p_2. Consequently, the price order is such that $p_1 \leq p_2$ and $p_3 \leq p_4$.

It is left to establish that $p_2 \leq p_3$. Let us first show that $p_2 \leq p_4$. Note that customers 2 and 4 cannot be the residual customer for L as customers 1 or 3 would be selected by the rationing rule, given that $p_1 \leq p_2$ and $p_3 \leq p_4$.

Suppose $p_4 < p_2$ and that the capacity constraint is binding for L (this occurs when L has lower prices than R for all customers). In this case either customer 4 or 2 is rationed (given $p_3 \leq p_4$ and $p_1 \leq p_2$). If customer 2 is rationed (which implies that L serves customer 4, but not 2), then it is profitable to increase p_4 to p_2 as this ensures that a higher price p_2 is realized at the lower costs for serving customer 2. If instead customer 4 is rationed, increasing p_4 has no effect on profits, whereas a lower p_4 strictly reduces profits, if it results in customer 2 being rationed (recall that $p_4 < p_2$ and costs are lower for customer 2). In summary, in this case there is a strict incentive to increase p_4 as long as it is not certain that customer 4 is rationed; once this is certain there is still a weak incentive.

Suppose $p_4 < p_2$ and that the capacity constraint of L is not binding. In that case increasing p_4 increases the expected profits of L. To see this, consider the marginal profit of L when changing p_4: As L faces equilibrium strategies of R which all have the property $p_4^R = p_3^R$, and $F_3^R = F_4^R$ and are designed such that L is indifferent over p_3 (the marginal profit (10) for $i = 3$ is zero), L has a strict incentive to increase p_4 for which it has larger costs (the marginal profit (10) for $i = 4$ is positive). Thus, there is an incentive to increase p_4 if it is below p_2 up to the point where it is certain that customer 4 is never served.

However, if it is certain that customer 4 is the one that is rationed in all situations where L is capacity constrained, the capacity constraint never binds for the first three customers. In that case p_3 is chosen according to the marginal condition. This condition ensures that L is indifferent over p_3 (the marginal profit (10) for $i = 3$ is zero), such that it is always a best response to increase p_3 up to p_2. This establishes that in response to the price distributions of the candidate equilibrium, it is not more profitable for L to set prices that are not weakly increasing. □

Proof of Lemma 6. The probability of an allocative inefficiency of c is the probability that $p_3^L < p_3^R$ or $p_2^R < p_2^L$. If L or R play a uniform price, as they do in the price interval from $[4c, v]$, the probability that the conditional probability of an inefficiency is 1. The probability that R or L will play a uniform price is 1 minus the probability that both will play prices in the lower range up to $4c$, where $\underline{F}_c(4c) = \underline{F}_d(4c)$ holds. Statistical independence of the mixed strategies of different players implies a probability of the event that at least one firm will plays uniform prices of

$$1 - \underline{F}_d(4c)^2 = 1 - \left(\frac{7c - v}{3c}\right)^2 .$$

With the complementary probability, both firms play prices in $\left[\underline{p}, 4c\right]$. These include increasing prices. In this interval, the probability for one of the inefficient cases, let us consider, $p_3^L < p_3^R$, is

$$\int_{\underline{p}}^{4c} \int_{\underline{p}}^{p_3^R} \underline{f}_d(p_3^L) \cdot \underline{f}_c(p_3^R) dp_3^L dp_3^R = \int_{\underline{p}}^{4c} \underline{F}_d(p_3^R) \cdot \underline{f}_c(p_3^R) dp_3^R$$

$$= \int_{\underline{p}}^{4c} \frac{3p - v - 5c}{3p - 9c} \cdot \frac{2(p - 2c) - \left(2p - \frac{2}{3}v - \frac{10}{3}c\right)}{(p - 2c)^2} dp = \int_{\underline{p}}^{4c} \frac{3p - v - 5c}{3p - 9c} \cdot \frac{\frac{2}{3}(v - c)}{(p - 2c)^2} dp.$$

By symmetry, the probability of an inefficiency is the same for customer 2 as for customer 3, yielding a probability of an inefficiency in the range where both firms play increasing prices of

$$2\int_{(v+5c)/3}^{4c} \frac{3p-v-5c}{3p-9c} \cdot \frac{\frac{2}{3}(v-c)}{(p-2c)^2} dp.$$

Overall, the probability of an inefficiency of c over the whole price range is

$$1-\left(\frac{7c-v}{3c}\right)^2 + 2 \cdot \int_{(v+5c)/3}^{4c} \frac{3p-v-5c}{3p-9c} \cdot \frac{\frac{2}{3}(v-c)}{(p-2c)^2} dp.$$

The first part is the probability that one of the two firms will play uniform prices. In this case an inefficiency occurs with probability 0.5 at customer 2 and 0.5 at customer 3. The second term is the probability that the more distant firm will offers the lower price to either customer 2 or 3 when both firms play increasing prices. The overall probability is strictly larger than $5/9 = \min_{v\in[5c,7c]}(1-(7c-v)3c)$, which is the minimal probability that at least one firms will play a uniform price vector with a price in $[4c, v]$. $\square$

Proof of Proposition 4. The existence of a symmetric mixed strategy equilibrium of the pricing game follows from Theorem 6 in Dasgupta and Maskin (1986). The conditions are met as the action space is a compact and a non-empty subset of the real numbers, the sum of the firms' pay-offs is continuous, individual pay-offs are bounded and weakly lower semi-continuous, with a strict inequality at the point of symmetry. Theorem 6 in Dasgupta and Maskin (1986) implies that all possible points of discontinuity in the pay-off functions are atomless. As every price can be a point of discontinuity, the whole price distribution must be atomless.

Note that the sum of the firms' pay-offs is $3\min(p_L, p_R) + \max(p_L, p_R) - 7c$ without subcontracting and $4\min(p_L, p_R) - 6c$ in the case of subcontracting. These profits are equal if the firms are indifferent between subcontracting and not subcontracting. Individual profits are weakly lower semi-continuous as they do not jump downwards within the price support, except in cases where the prices of both firms are equal.

Price v is played with positive density. Suppose to the contrary that there was an upper boundary $\overline{p} < v$ in the symmetric mixed strategy equilibrium while there are no mass points in the distribution functions. The profit of such a price is $\overline{p} - c < v - c$, such that it is profitable to move density from $\overline{p}$ to v. The profit at a price of v is the residual demand profit $v - c$, as the receiver of a subcontract obtains the associated rent. As the profit must be the same on the whole support, the equilibrium profit is $v - c$.

Not all efficient cross-supplies take place as the maximal price difference, i.e., the range of prices in the support of the price distributions, is larger than c. To see this, note that the lowest possible price without subcontracting is $\underline{p}$ as defined in (4). The lowest price with subcontracting is even lower as the profit at the upper bound is still $v - c$, and the profit at the lower bound is higher, due to the additional profits from subcontracting. The price difference $v - \underline{p}$ is thus a lower bound for the range with subcontracting. The price range without subcontracting is already larger than c given the assumption $v \geq 4c$ because $v - \underline{p} = \frac{1}{3}(2v - 5c) \geq \frac{1}{3}(8c - 5c) = c$. Thus, price differences larger than c occur with positive probability in the case of subcontracting. Although it would increase efficiency, subcontracting does not take place in these cases because the firms lose more in revenues than they gain in cost reductions. □

Given the atomless price distribution, cross-supplies take place with positive probability. Total surplus is higher as cross-supplies reduces the transport costs, whereas the expected profit per firm remains at $v - c$, equal to the profit without the efficiency increasing cross-supplies. As firms only engage in cross-supplies when their joint surplus increases, they must, on average, set lower prices for their expected profit to remain constant despite lower costs. This implies that customers benefit from cross-supplies through lower prices, on average – at least when the firm demanding the cross-supply gets the associated rent.

Proof of Proposition 5. The first part of the proof with respect to the existence of a symmetric mixed strategy equilibrium and the characteristics of the profit function is virtually identical to that of Proposition 4.

The highest price that is played with positive density is v. The expected profit at this price is bounded from above by $(v-c)+c = v$, where the first part is the residual demand profit and the second part the rent from subcontracting in the limit when the price of the competitor approaches v. The lower bound profit can be found by excluding the subcontracting profit, which yields additional profits for the firm with higher prices. The lower bound profit is thus $v - c$. The lowest price p' that is played must yield the same profit as the largest price v. To compute a lower bound for the price range, we obtain an upper bound of p' by equalizing the associated profit at this price with the upper bound profit at a price of v, that is, $v = 3p' - 6c \Leftrightarrow p' = v/3 + 2c$. Hence, the difference between the highest and the lowest price that are played with positive density is at least $v - p' = \frac{2}{3}v - 2c$, which is at least c if $v \geq 4.5c$. For this parameter range clearly not all efficient cross-supplies take place as the range of the support is larger than c. As subcontracting takes place with positive probability, total surplus increases as the average costs decrease. The increase in the total surplus is at most c, which is the efficiency gain if a subcontract is realized with probability 1. □

Appendix II: Explicit price strategy for strictly increasing prices

In this Appendix we present an example of an equilibrium price strategy for the case of strictly increasing prices ($5c < v < 7c$), as described in Proposition 3. In particular, we illustrate that a firm can draw prices from a joint distribution such that marginal distributions are F_c for the two closest and F_d for the two most distant prices, as defined in (8) and (7), and the resulting price vectors are always weakly increasing with $p_1 = p_2 \leq p_3 = p_4$.

Suppose that firm L initially draws a price $p_1 \in [\underline{p}, v]$ from the distribution function F_c. It then sets $p_2 = p_1$ as also p_2 must be played according to the marginal distribution F_c and $p_1 = p_2$ is a requirement of the equilibrium strategies.

Recall that $F_c(p)$ equals $F_d(p)$ for $p \in [4c, v]$, which is only consistent with uniform price vectors. Firm L thus also sets the other prices p_3 and p_4 equal to p_1 in this interval.

For $p_1 \in [\underline{p}, 4c)$, firm L faces the problem that only playing uniform prices is not consistent with the marginal distributions as F_d first-order stochastically dominates F_c in that range. In particular, there is a price $\tilde{p} \in (\underline{p}, 4c)$, such that

$$\begin{aligned} f_c(\tilde{p}) &= f_d(\tilde{p}), \\ f_c(p) &> f_d(p) \text{ for } [\underline{p}, \tilde{p}), \\ f_c(p) &< f_d(p) \text{ for } (\tilde{p}, 4c]. \end{aligned}$$

Small prices of close customers are played more often than small prices of distant customers. Firm L can now do the following:

1. For each realized price $p_1 \in [\underline{p}, \tilde{p})$,
   (a) with probability $\alpha \equiv f_d(p_1)/f_c(p_1)$ set uniform prices $p_1 = p_2 = p_3 = p_4$.
   (b) with probability $(1 - \alpha)$ set prices p_3 and p_4 in the interval $[\tilde{p}, 4c]$ according to the density function $r(p) = [f_d(p) - f_c(p)] \,/\, [F_d(4c) - F_c(4c) - (F_d(\tilde{p}) - F_c(\tilde{p}))]$.
2. For each realized price $p_1 \in [\tilde{p}, 4c]$,
   (a) with probability 1 set uniform prices $p_1 = p_2 = p_3 = p_4$.

In the case of low prices below $\tilde{p}$, the firm draws a higher price in the interval $[\tilde{p}, 4c]$ based on $r(p)$ with probability $(1 - \alpha)$ according to step 1 (b). This density function is constructed in a way that density for distant prices is allocated from any point in the lower interval $[\underline{p}, \tilde{p})$ to the upper interval $[\tilde{p}, 4c)$ in proportion to the density $f_d(p) - f_c(p)$. This is the "missing" density when only uniform prices are played in response to realizations of $p_1 \in [\tilde{p}, 4c]$ with probability α according to step 2 (a) of the above rule. Consequently, the distant prices materialize according to the marginal distribution function F_d. Note that the firm only plays strictly increasing prices in step 1(b), which happens with probability $\int_{\underline{p}}^{\tilde{p}} f_1(p)(1 - \alpha)dp = F_1(\tilde{p}) - F_3(\tilde{p})$. In summary, the joint distribution is characterized as follows: In the upper part of the interval starting at $4c$, only uniform price vectors are played. Uniform prices are also often played in the lower part of the interval (based on step 1 (a)). In order to ensure that the different marginal densities f_c and f_d materialize in the lower interval $[\underline{p}, 4c)$, strictly increasing prices are generated in step 1 (b) of the above rule.

Appendix III: Unexplained excess margins when using an over-simplified model

Consider a market with customer-firm-specific transport costs and pricing as well as capacity constraints. Without the Bertrand-Edgeworth model developed in this article, one would potentially use a pre-existing model of competition to predict the competitive market outcome. Such a traditional model would disregard either capacity constraints or heterogeneous transport costs (as done in the modeling of the European Commission in recent merger cases, see Section 9). However, as we demonstrate below, these simplified models may predict competitive prices and profits, which are significantly below those which the more appropriate model would predict. Thus, by disregarding one of these factors, a competition authority may wrongly conclude that the observed market outcome cannot derive from competition, but rather from coordination.

Model	Predicted profit	Unexplained excess margin ($v = 7c$)
Model of this article (no subcontracting)	$v - c$	0% (this reflects the "true" market outcome with capacity constraints and transport costs)
Bertrand-Edgeworth with uniform prices and uniform costs, based on accounting costs	$v - 1.75c$	$(v - c)/(v - 1.75c) - 1 = 14\%$
Bertrand-Edgeworth with uniform prices and uniform costs based on simple average costs	$v - 2.5c$	$(v - c)/(v - 2.5c) - 1 = 71\%$
Asymmetric Bertrand competition with unlimited capacity	$4c$	$(v - c)/4c - 1 = 50\%$

Table 4: Profit predictions of various models and excess margins relative to model with heterogeneous costs and capacity constraints (first line).

Table 4 lists the equilibrium profits implied by the various models. *Main model* refers to the model analyzed in Section 5.3, which accounts for firm-customer-specific transport costs as well as capacity constraints and allows for customer-specific prices. *Unexplained excess margin* is the fraction of the predicted profit of the main model over the prediction of the respective simplified model, minus one. Consider that one observes market data which is as predicted by the main model (that is assumed to appropriately reflect the market). The higher the actual margins are compared to those of a more simple competitive model, the more likely one would conclude that the market outcome is not the result of competition, but may rather result from coordination. In the example depicted in Table 4, the actual competitive margins may be between 14% to 71% above the margins predicted by the Bertrand-Edgeworth model with uniform costs, depending on the employed cost measures in the more simple model.

Let us briefly sketch how we calculated the results presented in the table. We computed the unexplained excess margin as the ratio of the profit of the main model (assumed to reflect the "observed true" market outcome) and the profit of the simplified model, minus one. This measures how much the "observed true" margin is above the margin that can be explained with a (traditional) competitive model. Note that we use our main model without subcontracting here. With subcontracting, the predicted profits of our model can be even higher and can range from $v - c$ to v. When subcontracting takes place but is not accounted for, the unexplained excess margins can thus be even higher.

Let us now explain the origin of the expressions in the different lines. The margin $(v - c)$ is taken from the the main model (see Proposition 2). The margin range in case of subcontracting is taken from Propositions 4 and 5.

For the Bertrand-Edgeworth models with uniform prices and costs, note that the residual profit, and thus, equilibrium profit, is $v - k$, where k denotes the uniform costs of serving any customer. Here, we consider two variants of how these "uniform" costs are computed in such a way that the true costs differ across customers:

1. Assuming that the actual market outcome is as characterized in Proposition 2 yields accounting costs (as in the firm's management accounts) per unit of $k = (c + 2c \cdot 50\% + 3c \cdot 50\%)/2 = 1.75c$ for $v \geq 7c$;

2. Using simple average costs across all customers yields $k = (1c + 2c + 3c + 4c)/4 = 2.5c$. The implicit assumption is here that each customer is served by the same firm with the same probability.

For asymmetric Bertrand competition without capacity constraints, the profit is taken from the reference case in Section 4.2.

Appendix IV: Decreasing prices

Proposition 6. *There exists no equilibrium in which the firms only play prices that (weakly) decrease in distance (such as $p_1^L \geq p_2^L \geq p_3^L \geq p_4^L$ for firm L), except for uniform prices (that is $p_1^j = p_2^j = p_3^j = p_4^j$, $j \in \{L, R\}$).*

Proof of Proposition 6. Suppose that firm R plays strictly decreasing price vectors of the form $p_1^R < p_2^R < p_3^R < p_4^R$. Let us first establish that price vectors with (weakly) decreasing prices and a strict decrease at the prices for customers 3 and 4 cannot be a best response of firm L. Let us consider all possible relations between the prices of L and R: In the following, when we speak of a price *below (above)* another price, we mean weakly (strictly) for customers 1 and 2 and strictly (weakly) for customers 3 and 4 which ensures that the tie-breaking rule of lower costs has the intuitive result that the firm that offers the ‚lower' price serves the customer as long as it has sufficient capacity.

- For each customer, the price of L is below that of R. L gets the three most distant customers (2, 3, and 4). For L, raising p_4^L to the level of p_3^L is profitable. This price increase ensures that L still has the lowest price for all customers and customer 1 is still rationed. Hence, the only effect of an increase of p_4^L to the level of p_3^L is an increase in the margin earned on customer 4.

- For each customer, price of L is above that of R. L only gets residual demand from customer 4. L is weakly better off by raising p_4^L to the level p_3^L: Serving customer 4 is least profitable for L. If customer 4 was the residual demand segment before the price change and now it is a different customer, this is profitable. If customer 4 remains the residual demand segment, the increase is also profitable.

- For customer 1, the price of L is above the price of R; for the other customers the price of L is below the price of R. Firm L gets customers 2, 3, and 4. Raising the price p_4^L to the level p_3^L cannot make firm L worse off: Firm L still gets the same customers, and customer 4 at a higher margin.

- For customers 1 and 2, the price of L is above the price of R; for customers 3 and 4 price of L is below price of R. Firm L gets customers 3 and 4, firm R customer 1 and 2. Raising the price p_4^L to the level p_3^L cannot make firm L worse off: Firm L still gets the same customers, and customer 4 at a higher margin. Note that the initial prices may be the same for customer 2 ($p_2^L = p_2^R$). In that case firm L also serves customer 2, due to cost minimization. The previous logic for raising p_4^L does not change.

- For customers 1, 2, and 3, the price of L is above the price of R; for customer 4 the price of L is below the price of R. Firm L only gets customer 4, firm R customers 1, 2 and 3. Raising the price p_4^L to the level p_3^L cannot make firm L worse off: It may happen that now all prices of firm L are above those of firm R, but firm L still gets customer 4, albeit now at a higher margin.

In summary, a best response to strictly decreasing price vectors of the other firm can at most be a decreasing price vector where the prices for the two most distant customers are equal.

In a next step, assume that firm R plays price vectors of the form $p_1^R = p_2^R \leq p_3^R < p_4^R$. Let us check whether a best response can be a price vector where the price for customer 1 is strictly above the price for customer 2, and in particular, $p_1^R > p_2^R \geq p_3^R = p_4^R$. Let us again consider all possible relations between the prices of L and R and check whether switching the prices for customers 1 and 2 is profitable for L.

- For each customer, the price of L is below that of R . L gets three customers: 2, 3, and 4. L is strictly better off by switching the prices for customers 1 and 2. As R charges these customers the same price and as the initial price of L for customer 2 is strictly below that for customer 1, the price switch implies that firm L now serves customers 1, 3, and 4 instead for 2, 3, and 4 (rationing that maximizes customer surplus). Firm L now has the same revenue but strictly lower costs, which means that the switch is profitable.

- For each customer, the price of L is above that of R . L only gets one customer as residual demand. L would be better off by switching the prices for customers 1 and 2. As R charges these customers the same price and as the initial price of L for customer 2 is strictly below that for customer 1, rationing implies that firm L never served customer 1 before the price switch. With the switch, rationing might allocate customer 1 to firm L, yielding strictly lower costs and at least the revenue obtained from any other customer that might have been allocated as a result rationing to firm L before the switch. This is profitable

- For customer 1, the price of L is above the price of R; for the other customers price of L is below price of R. There is no rationing. Firm L gets customers 2, 3, and 4. Firm

L switching prices for customers 1 and 2 implies that it serves customer 1 instead of customer 2 with the same revenue and at strictly lower costs. This is profitable.

- For customers 1 and 2, the price of L is above the price of R; for customers 3 and 4 price of L is below price of R. Firm L gets customers 3 and 4, firm R customer 1 and 2. Firm L switching prices for customers 1 and 2 has no effect on the allocation of customers and thus profits. Note that the initial prices may be the same for customer 2 ($p_2^L = p_2^R$). In that case firm L also serves customer 2, due to cost minimization. In this case, it is even profitable for firm L to switch prices for customers 1 and 2 as it then serves customer 1 instead of 2, which reduces its costs.

- For customers 1, 2, and 3, the price of L is above the price of R; for customer 4 the price of L is below the price of R. There is no rationing. Firm L gets customer 4, firm R customers 1, 2, and 3 (if the prices for 2 and 3 are different). In this case switching prices 1 and 2 has no effect for firm L. If the prices 2 and 3 of both firms are the same, firm L gets customers 2 and 4, firm R customers 1 and 3. In this case, firm L switching prices of customers 1 and 2 increases profit as it serves 1 instead of 2 as a consequence. If the prices of both firms are the same only for customer 3 are the same, firm L only gets customer 4 and switching prices of customers 1 and 2 has no effect. Overall, switching prices is weakly profitable.

In summary, a best response to decreasing price vectors of firm R of the form $p_1^R = p_2^R \leq p_3^R < p_4^R$ cannot have decreasing prices of firm L between customers 1 and 2 (it must be that $p_1^L \geq p_2^L$). Moreover, as previously shown, it must be that in a best response the prices for the two most distant customers are at least equal ($p_4^L \geq p_3^L$). This leaves us with price vectors of the form $p_1^L = p_2^L \geq p_3^L = p_4^L$ as possible decreasing price vectors (and analogously for firm R). In the next step, we show that there exists no equilibrium in which both firms play such price vectors.

Intuitively, such decreasing price vectors cannot form a symmetric equilibrium with mixed strategies because this requires that a firm is simultaneously indifferent over their prices for the two closest and the two most distant customers, although it has higher costs for the more distant customers and faces higher prices of the competitor there (the other firm would also play decreasing prices in a symmetric equilibrium). This would yield an incentive to charge the distant customers strictly higher prices – which contradicts decreasing prices.

To see this contradiction formally, consider the marginal incentive to increase prices for firm L. For all prices played with positive density, there must be no incentive to do so, that is, for any $p_i \in [\underline{p}, v]$, denoting the cost for serving customer i by C_i

$$(p_i - C_i)\, f_i^R(p_i) + F_i^R(p_i) = 0.$$

Integrating both sides over p_i, with upper limit b with $\underline{p} < b \leq v$, yields

$$\int_{\underline{p}}^{b} (p_i - C_i)\, f_i^R(p_i) dp_i + \int_{\underline{p}}^{b} F_i^R(p_i) dp_i = 0.$$

This equation can be simplified using integration by parts on the first term to obtain

$$(b - C_i)\, F_i^R(b) - \int_{\underline{p}}^{b} F_i^R(p_i) dp_i + \int_{\underline{p}}^{b} F_i^R(p_i) dp_i = 0,$$

which yields

$$(b - C_i)\, F_i^R(b) = 0.$$

Comparing the condition for k and i with $k > i$ yields a contradiction. Note that decreasing prices played by firm R imply $F_i^R(p) \geq F_k^R(p)$ if $k > i$. Both conditions, for i and k cannot be satisfied simultaneously as $C_i < C_k$ and $F_i \geq F_k$. □

Appendix V: Endogenous capacities with price discrimination

First, let us discuss the different capacity configurations step by step. To show that $l = 2$ and $r = 2$ are still an equilibrium, it is sufficient to derive and compare the market outcomes with capacities of ($l = 3$, $r = 2$), $(2, 2)$, $(2, 1)$, and $(4, 2)$.

For the capacity levels $(3, 1)$, $(2, 1)$, and $(2, 2)$, monopoly prices result as the total capacity is less than demand.

For $(3, 2)$, there is no pure strategy equilibrium, but the mixed strategy equilibrium in uniform prices persists even when price discrimination is possible. To see this, we apply the same logic as before. First, suppose both firms play increasing prices. Consider that R plays uniform prices. The best response in increasing prices by L is to set $p_1 = p_2 = p_3$. The price p_4 is again not relevant for the profits as long as all firms play only increasing prices. Note that firm L always serves customers 1 and 2 if its prices p_1 and p_2 do not exceed its prices p_3 and p_4. As firm L never serves customer 4, L is indifferent over p_4 and thus uniform prices are a best response. Similarly, if R faces L playing only uniform prices and R chooses weakly increasing prices, R never serves customers 1 and 2 (its most distant customers), which makes R indifferent over the prices for these customers. Let us now construct an equilibrium in uniform prices. In equilibrium, a uniform price distribution played by L must be such that R is indifferent over changing p_3 and p_4 simultaneously. This gives R an incentive to reduce p_3 and increase p_4 as R serves customer 4 with certainty. This provides for a strict incentive to increase the price p_4 marginally, as long as $p_4 \leq p_3$ and $p_4 \leq v$. Hence, it is a best response of R to charge uniform prices $p_1 = p_2 = p_3 = p_4$, where the first two equalities follow from the indifference of R over prices for customers that it serves with zero probability, while the last equality follows from the argument presented before. As there is no incentive for any firm to deviate from uniform prices with an individual price, this equilibrium exists for the same parameters as in the case that only uniform prices are allowed. Hence, this equilibrium exists for the whole parameter range. The equilibrium has the same profits as above ($2v - 3c$ for firm L, $\frac{4}{3}v - c$ for R). One intuition why there are no strictly increasing price is that firms “compete” for only one customer while all other customers are either in their home market or never reached. Due to the logic of efficient rationing, home market prices are fixed by the price level for the customer over which firms compete.

Table 5 contains the resulting profits for the case of $v > 5c$. The case $v \leq 5c$ only differs in that for the capacity configurations (3,3) and (4,3) standard Bertrand pure strategy equilibria exist with a profit of $4c$ for each firm.

In summary, there is no profitable deviation by one capacity unit from the case $(2, 2)$. Note that in the case of capacities of (4,2) the profits for L are identical to the case (3,2) as long as there is an equilibrium where profits are defined by the residual profit of the firm with a larger capacity. This is, for instance, the case in a mixed strategy equilibrium with increasing prices. Hence, there would be no incentive to increase capacity starting from the case (2,2), making this Nash equilibrium of the capacity game.[28]

		Firm R		
	Capacities	1	2	3
	1	$v-c,\ v-c$	$v-c,\ 2v-3c$	$v-c,\ 3v-6c$
Firm L	2	$2v-3c,\ v-c$	$2v-3c,\ 2v-3c$	$\frac{4}{3}v-c,\ 2v-3c$
	3	$3v-6c,\ v-c$	$2v-3c,\ \frac{4}{3}v-c$	$v-c,\ v-c$

Table 5: Profits for different capacity levels when total demand is four and there is price discrimination, case $v > 5c$.

[28]Note that we do not explicitly construct an equilibrium for the case (4,2). It can be shown that in this case a mixed strategy equilibrium in uniform prices does not exist for any parameter constellation.

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